

## 國立臺北科技大學

九十七學年度四年制二、三年級轉學生招生考試

系所組別：四技三年級化學工程與生物科技系

## 第二節 專業科目（一）工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) A curve with Cartesian equation  $y=f(x)$  passes through the origin.

Lines drawn parallel to the coordinate axes through an arbitrary point of the curve form a rectangle with two sides on the axes. The curve divides every such rectangle into two regions  $A$  and  $B$ , one of which has an area equal to  $n$  times the other. Find the function  $f$ .

2. (15%) Solve the given differential equation by using an appropriate substitution.

$$\frac{dy}{dx} = \sin(x+y)$$

3. (15%) Use Green's theorem to compute the work done by the force field

$$\vec{F}(x,y) = (y+3x)\vec{i} + (2y-x)\vec{j} \text{ in moving a particle once around the ellipse}$$

$$4x^2 + y^2 = 4 \text{ in the counterclockwise direction.}$$

4. (15%) Solve the linear system

$$\frac{dx}{dt} = 5x + 4y$$

$$\frac{dy}{dt} = x + 2y$$

subject to the initial conditions  $x(0)=2, y(0)=3$ .

5. (10%) Find the Fourier cosine and sine series of the function

$$f(x)=1, 0 \leq x \leq 2.$$

6. (15%) Using Laplace transform, find the solution of the initial value problem

$$x \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = xt \quad u(x,0)=0, \quad u(0,t)=t$$

7. (15%) Determine  $a, b, c, d, e, f$ , given that the vectors  $(1,1,1)^T$ ,

$(1,0,-1)^T$ , and  $(1,-1,0)^T$  are eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ d & e & f \end{bmatrix}$$