

國立臺北科技大學

九十三年學年度電腦通訊與控制研究所碩士在職專班入學考試

乙組：線性代數 試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共【五】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。

- 一、(40%) Answer each of the following as true (T) or false (F). If your answer is false (F), please specify why it is wrong.
- (1) An $n \times n$ matrix A has an inverse if and only if A has rank n .
 - (2) An $n \times n$ matrix is diagonalizable if and only if it has n linearly independent eigenvectors.
 - (3) If A is an $n \times n$ matrix, then $|2A|=2|A|$.
 - (4) If a subset of a vector space does not include the zero vector, that subset cannot be a subset.
 - (5) If A and B are $n \times n$ matrices and they are invertible, then $(AB)^{-1} = A^{-1}B^{-1}$
 - (6) If $\langle x, y \rangle$ means the inner product of two vectors x and y in R^n , then $\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
 - (7) The vectors $(1, 0, 1)$ and $(-1, 1, 0)$ are orthogonal.
 - (8) If x and y are eigenvectors of A associated with the different eigenvalues λ_1 and λ_2 , then $x+y$ is an eigenvector of A associated with eigenvalue $\lambda_1+\lambda_2$.
 - (9) $S = \{(1, -5, 4), (11, 6, -1), (2, 3, 5)\}$ is a basis for R^3
 - (10) If two vectors are orthogonal with respect to one basis, then they are orthogonal with respect to any other basis.

二、(10%) Suppose an $n \times n$ matrix A satisfies the equation $A^2 - 2A + I = 0$, where I is an identity matrix. Show that $A^3 = 3A - 2I$ and $A^4 = 4A - 3I$.

三、(15%) Solve the linear system $Ax = b$ for x , when

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 4 \\ -3 \\ -2 \\ -2 \end{bmatrix}$$

四、(15%) Find a , if

$$\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = 0, \text{ and } a \text{ is a real number.}$$

五、(20%) $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

- find eigenvalues of A .
- If A is diagonalizable and can be expressed as $A = PDP^{-1}$, find P and D .
- Using the results of (b) to calculate A^{10} .