

# 國立臺北科技大學九十九學年度碩士班招生考試

系所組別：3721、3722 有機高分子研究所乙組

## 第一節 工程數學 試題

第一頁 共一頁

### 注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following equations

(a)  $x^2 y'' = (y')^2 + 2xy'$  (10%)

(b)  $x^2 y dx - (x^3 + y^4) dy = 0$  (10%)

2. Find the general solution of the following ODE

(a)  $y'' - 9y = 4 + 5 \sinh 3x$  (10%)

(b)  $(D-1)^2(D+1)y = x^2 e^x$  (10%)

(c)  $(D^3 - D)y = \frac{1}{1+e^x}$  (10%)

3. Solve the simultaneous linear differential equations

$$\frac{d^2 x}{dt^2} - x - 2y = t$$

$$\frac{d^2 y}{dt^2} - 2y - 3x = 1 \quad (10\%)$$

4. Solve the initial value problem by Laplace transforms

$$ty'' + (4t-2)y' - 4y = 0, \quad y(0) = 1 \quad (10\%)$$

5. Two tanks are connected by a series of pipes as shown in Figure P4.

Tank 1 initially contains 60 gallons of brine in which 11 pounds of salt are dissolved. Tank 2 initially contains 7 pounds of salt dissolved in 18 gallons of brine. Beginning at time zero a mixture containing  $\frac{1}{6}$  pound of salt for each gallon of water is pumped into tank 1 at the rate of 2 gallons per minute, while salt water solutions are interchanged between the two tanks and also flow out of tank 2 at the rates shown in the diagram. Four minutes after time zero, salt is poured into tank 2 at the rate of 11 pounds per minute for a period of 2 minutes. Determine the amount of salt in each tank for any time  $t \geq 0$ . (15%)

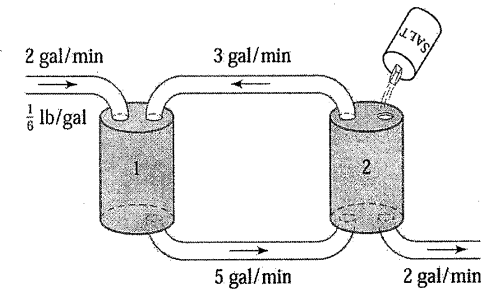


Figure P4

6. Find all the eigenvalues and eigenfunctions for the following BVP

$$x^2 y'' + 3xy' + \lambda y = 0 \quad y(1) = 0 \quad y(2) = 0 \quad (15\%)$$