

國立臺北科技大學九十九學年度碩士班招生考試

系所組別：1111、1112、1120 機電整合研究所甲、乙組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、A particle with mass m is ejected vertically from the ground, where the acceleration of gravity is g . The initial velocity is v_0 . The drag due to the air friction is assumed to be proportional to its momentum mv and can be written as kmv .

1. show that the equation of motion of the particle is

$$\frac{dv}{dt} = -g - kv. \quad (5\%)$$

2. Solve the velocity function $v(t)$. (5%)

3. Find the maximum height that the particle will reach. (10%)

二、Solve the general solutions of $x(t)$ and $y(t)$ in the following simultaneous differential equations:

$$\begin{cases} \frac{d^2x}{dt^2} - x - 2y = t \\ \frac{d^2y}{dt^2} - 2y - 3x = 1 \end{cases} \quad (20\%)$$

三、By using Laplace transform, solve the function $x(t)$ in the following simultaneous equations:

$$\begin{cases} \dot{x} + x + 3 \int_0^t y dt = \cos t + 3 \sin t \\ 2\dot{x} + 3\dot{y} + 6y = 0 \end{cases}$$

with the associated initial conditions $x(0) = -3$, $y(0) = 2$. (20%)

四、Determine the integral $\oint_C (y^2 dx + x^2 dy)$ where C is the boundary contour of the domain defined by a triangle with vertices at $(0,0)$, $(1,0)$, $(0,2)$. (20%)

Hint: Green theorem $\iint_A (\nabla \times \vec{\phi}) \cdot \vec{n} dA = \oint_C \vec{\phi} \cdot d\vec{S}$

$$\text{五、} [A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

1. Determine the eigenvalues and corresponding normalized eigenvectors of $[A]$. (5%, 10%)
2. Show that the eigenvectors are orthogonal. (5%)