注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 各題作答時均需列示計算過程，否則不予計分。

Note:

Standard Normal: $P(Z < 1.25) = 0.8944$, $P(Z < -1.25) = 0.1056$, $P(Z < 2.5) = 0.9938$,

$P(Z < -2.5) = 0.0062$, $P(Z < -1.96) = 0.025$, $P(Z < -1.645) = 0.05$;

t Distribution: $t_{0.1}(8) = 1.397$, $t_{0.1}(9) = 1.383$, $t_{0.05}(8) = 1.86$, $t_{0.05}(9) = 1.833$;

Chi-Square: $\chi^2_{0.025}(9) = 19.02$, $\chi^2_{0.025}(10) = 20.48$, $\chi^2_{0.05}(9) = 16.92$, $\chi^2_{0.05}(10) = 18.31$;

F Distribution: $F_{0.05}(2,12) = 3.89$, $F_{0.05}(2,14) = 3.74$, $F_{0.05}(3,12) = 3.49$, $F_{0.05}(3,14) = 3.34$.

1. Suppose that you want to invest $20,000 in the stock market by buying shares in one of two companies: A and B. Shares in Company A, though risky, could yield a 40% return on investment in a "bull" market during the next year. If the stock market conditions are not favorable, the stock may lose 20% of its value. Company B provides safe investments with 10% return in a "bull" market and only 5% in a "bear" market. The prior probabilities of 0.7 and 0.3 of a "bull" and a "bear" market were determined from available financial publications, respectively.

  (1) Where should you invest your money? **(5%)**

  (2) Suppose that rather than relying solely on these publications, you have decided to conduct a more "personal" investigation by consulting a friend who has done well in the stock market. The friend offers the general opinion of "for" or "against" investment. This opinion is further quantified in the following manner: If it is a "bull" market, there is a 80% chance the vote will be "for". If it is a "bear" market, the chance of a "for" vote is lowered to 40%.

  (a) If the friend's recommendation is "for," would you invest in stock A or in stock B? **(5%)**

  (b) If the friend's recommendation is "against," would you invest in stock A or in stock B? **(5%)**

  (c) Consider the expected value of sample information, if the friend would like to charge $100 for consulting fees, would you pay it? **(10%)**

2. A population has three elements {2, 5, 8}. Assuming that you sample with replacement, select all possible samples of n=2 from the population and $\overline{X} = \frac{X_1 + X_2}{2}$ where X_1 and X_2 represent the outcomes of the first and the second selection, respectively.

  (1) Construct the sampling distribution of the mean. **(5%)**

  (2) Find $Var(\overline{X})$. **(5%)**

3. The initial salary of NTUT graduates is normally distributed, with a mean of NT$ 35,000 and a standard deviation of NT$4,000. If you select a random sample of 25 employed graduates graduated from NTUT this year, please answer the following questions.

  (1) What is the probability that the sample mean of the initial salary between NT$33,000 and NT$ 37,000? **(10%)**

  (2) The probability is 0.95 that the sample mean of the initial salary will be between which two values (symmetrically distributed around the mean)? **(10%)**

4. A researcher wants to study the average number of people for each family in Taipei. A random sample of 9 families is selected and the numbers of people for the families are listed as follows:

  {3, 2, 3, 3, 2, 4, 5, 3, 2}.

Suppose the number of people for each family is normally distributed, please construct a 90% confidence interval estimate for the mean number of people for each family. **(10%)**

5. Suppose the volume of salad oil in a bottle is normally distributed for some production line. When the variance of the volume is greater than 1 liter, the production procedure is unstable. If you select a random sample of 10 bottles for testing from a supermarket and get the sample standard deviation 1.5 liters, can you say that the production procedure is unstable at the 0.05 level of significance? **(10%)**

6. A second-order autoregressive model for average mortgage rate is:

  $\text{Rate}_i = -1.8 + 1.5(\text{Rate})_{i-1} - 0.4(\text{Rate})_{i-2}$.

If the average mortgage rate in 2008 was 7.2, and in 2006 was 6, please forecast the average mortgage rate for 2010. **(10%)**

7. The retailing manager of a supermarket chain wants to determine whether store location has any effect on the sales volume of the new beverage product. Three different locations are considered: Taipei, Taichung, and Kaohsiung. The following provides 5 days sales volumes (in thousands of bottles) of the product for the three stores.

Taipei store: 10, 11, 13, 15, 17;

Taichung store: 8, 10, 11, 12, 14;

Kaohsiung store: 6, 8, 9, 13, 15.

(1) Please show the ANOVA table. **(10%)**

(2) At the 0.05 level of significance, is there evidence of a significant difference in mean sales among the various store locations? **(5%)**