# 國立臺北科技大學九十九學年度碩士班招生考試

## 系所組別：2230 電腦與通訊研究所丙組

## 第一節　電磁學　試題



**注意事項：**
1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. An element shown in Figure 1 below is defined by the following surfaces:

- $r = r_1$ and $r = r_2$,
- $\phi = 0$ and $\phi = \alpha$,
- $z = 0$ and $z = \ell$.

Compute the following if the material of the element is characterized by a permittivity of $\epsilon$:

1. The capacitance of the element if the surface at $\phi = 0$ has $V = 0$ and the surface at $\phi = \alpha$ has $V = V_0$. Then, determine the resistance of this element if the material is characterized by a conductivity of $\sigma$. Neglect fringing. (10%)
2. The capacitance of the element if the surface at $r = r_1$ has $V = 0$ and the surface at $r = r_2$ has $V = V_0$. Then, determine the inductance of this element if the material is characterized by a permeability of $\mu$. Neglect fringing. (10%)

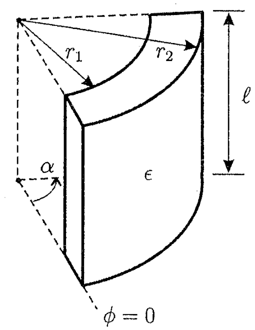


Figure 1.

二. Determine the electric field intensity **E** at a remote location $P(r, \theta, \phi)$ contributed by the following arrangements:

1. The electric dipole defined by its dipole moment $\mathbf{P} = qd\hat{x}$ shown in Figure 2(a). (8%)
2. The electric dipole defined by its dipole moment $\mathbf{P} = qd\hat{y}$ shown in Figure 2(b). (7%)

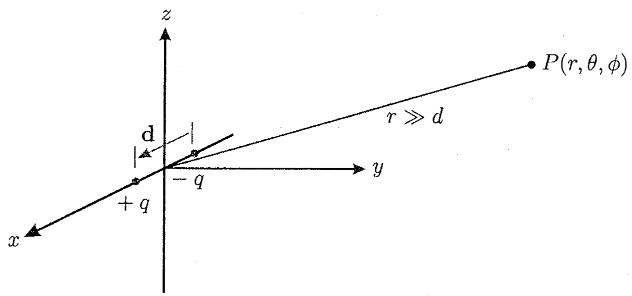


Figure 2(a).

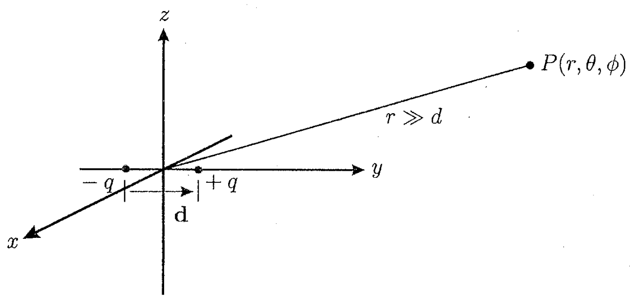


Figure 2(b).

三. Show a transmission line satisfying the following condition has a distortionless or non-dispersive nature. (15%)

$$\frac{R}{L} = \frac{G}{C}.$$

注意：背面尚有試題

四. In a source-free region, an electric wave $\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}} = \mathbf{E}_0 e^{-jk\hat{k}\cdot\mathbf{r}}$ propagates in $\hat{k}$ direction, where

- $\mathbf{k}$ is the wavenumber vector given by $\mathbf{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$,
- $\mathbf{E}_0$ is a constant vector,
- $\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the position vector.

Show the following:

1. The electric filed $\mathbf{E}$ satisfies the homogeneous Helmholtz's equation. (5%)
2. The electric filed $\mathbf{E}$ is a uniform plane wave. (8%)
3. The electric filed $\mathbf{E}$ and the magnetic field $\mathbf{H}$ are perpendicular to each other and both are normal to the propagation direction $\hat{k}$. (7%)

五. The definition for the magnetization vector $\mathbf{M}$ of a magnetic material is given by

$$\mathbf{M} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{N\Delta v} \mathbf{m}_i}{\Delta v},$$

where

- $\Delta v$ is the volume of the magnetic matrial,
- $\mathbf{m}_i$ is the individual magnetic dipole within $\Delta v$,
- $N$ is the number of magnetic dipoles per unit volume.

Derive in detail the expressions for the equivalent magnetization current densities $\mathbf{J}_m$ and surface current densities $\mathbf{J}_{ms}$ of the magnetic material in terms of $\mathbf{M}$. (15%) The vector magnetic potential established by a magnetic dipole $\mathbf{m}$ can be expressed as

$$\mathbf{A} = \frac{\mu_0 \mathbf{m} \times \hat{r}}{4\pi r^2},$$

where $r$ is the distance from the center of the loop to the point where $\mathbf{A}$ is to be determined and $\hat{r}$ is the unit vector in that direction.

六. A metal bar slides over a pair of conducting rails in a uniform magnetic field $\mathbf{B} = \hat{z}B_0$ with a constant velocity $\mathbf{u}$, as shown in Figure 3.

1. Determine the open-circuit voltage $V_0$ that appears across terminals 1 and 2. (5%)
2. Assuming that a resistance $R$ is connected between the terminals, find the electric power dissipated in $R$. (3%)
3. Show that this electric power is equal to the mechanical power required to move the sliding bar with a velocity $\mathbf{u}$. Neglect the electric resistance of the metal bar and of the conducting rails. Neglect also he mechanical friction at the contact points. (7%)

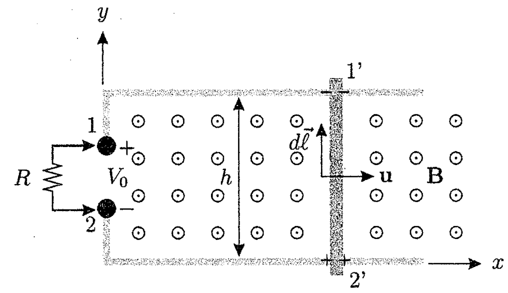


Figure 3.

Laplace's equation in cylindrical coordinates:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

and in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0.$$