國立臺北科技大學九十九學年度碩士班招生考試

系所組別：2220 電腦與通訊研究所乙組

第一節　工程數學　試題

注意事項：

1. 本試題共 7 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (20%) Consider a pair of random variables X and Y with joint probability mass function $P(X=x, Y=y)$ listed in the following table.

   A. Are X and Y independent? Explain your answer. (10%)

   B. Are X and Y uncorrelated? Explain your answer. (10%)

		x		
	$p(x,y)$	-1	0	1
	-1	1/6	0	1/6
y	0	0	1/3	0
	1	1/6	0	1/6

2. (20%) Let U_1 and U_2 be independent random variables both uniformly distributed in the interval [0,2]. Let $X = U_1$ and $Y = U_1 + U_2$.

   A. Find the conditional probability density function $f_{Y|X}(y \mid x)$. (7%)

   B. Find the conditional expectation $E[Y \mid X = x]$. (7%)

   C. Find the expectation $E[Y]$. (6%)

3. (10%) Let X be a random variable uniformly distributed in the interval [0,1]. Find the transformation $y = u(x)$ such that the new random variable $Y = u(X)$ has the following probability density function

$$f_Y(y) = 2y \cdot e^{-y^2}, \quad y \geq 0.$$

4. (10%) Consider a pair of Bernoulli random variables X and Y. Let X has the probability mass function in (1), and the conditional probability mass function of Y given X is listed in (2). Find the conditional probability mass function $P(X=1 \mid Y=0)$.

$$P(X=x) = \begin{cases} 0.6 & x=1 \\ 0.4 & x=0 \end{cases} \qquad (1)$$

$$\begin{aligned} P(Y=1 \mid X=1) &= 0.7 \\ P(Y=0 \mid X=1) &= 0.3 \\ P(Y=1 \mid X=0) &= 0.8 \\ P(Y=0 \mid X=0) &= 0.2 \end{aligned} \qquad (2)$$

5. (20%) Consider the following linear transformation $T: R^3 \to R^4$.

$$T(x,y,z) = (x+2y-3z,\ -2x+4y+6z,\ 2y-z,\ -4y+2z)$$

   A. Is it one-to-one? Explain your answer. (10%)

   B. Is it invertible? Explain your answer. (10%)

6. (10%) Let U and V be vector spaces and $T: U \to V$ be a linear transformation. Prove that the range of T is a subspace of V.

7. (10%) The planes $x+2y-2z=3$ and $2x+4y-4z=7$ are parallel. Find the distance between these planes.