

國立臺北科技大學九十九學年度碩士班招生考試

系所組別：2111、2112、2120、2130

電機工程系碩士班甲、乙、丙組

第二節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Consider the differential equation:  $y'' + 4y' + 4y = f_1(x) + f_2(x)$ .
  - (a) (10%) If  $f_1(x) = e^{-2x}$  and  $f_2(x) = 0$ , find the general solution.
  - (b) (5%) If  $f_1(x) = 0$  and  $f_2(x) = \delta(t-2)$ , find the particular solution.
  - (c) (5%) If  $f_1(x) = e^{-2x}$  and  $f_2(x) = \delta(t-2)$ , find the general solution.
2. For the Euler's equation  $Ax^3y''' + x^2y'' + 3xy' + 2y = x$ ,  $A \in R$ , please answer the following questions.
  - (a) (10%) If  $A = 0$ , find the general solution of the differential equation.
  - (b) (5%) If  $A \neq 0$ , find the characteristic equation of the Euler's equation after using the transformation  $t = \ln(x)$ .
3. (15%) Let  $\mathcal{L}[f(t)] = F(s)$  be the Laplace transform of  $f(t)$  and suppose that  $F(s)$  is differentiable. Find the inverse Laplace transform of  $F(s) = \tan^{-1}\left(\frac{\alpha}{s}\right)$ .

4. For the matrix  $A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 2 \\ 0 & 2 & 4 \end{bmatrix}$ , please answer the following questions.

- (a) (5%) Find the eigenvalues of  $A$ .
- (b) (10%) Calculate the matrix  $P$  such that  $P^TAP$  is diagonalized and  $P^T = P^{-1}$ .

5. Consider the matrix  $A = \begin{bmatrix} -3 & 0 & 2 & 0 & 1 \\ 0 & 3 & 0 & 5 & 0 \\ -2 & 0 & 1 & 2 & 0 \end{bmatrix}$ .

- (a) (10%) Find the range space  $R(A)$  and  $\dim(R(A))$ .
- (b) (10%) Find the null space  $N(A)$  and  $\dim(N(A))$ .

6. (15%) For the matrix  $A(t) = \begin{bmatrix} 1 & t^2 \\ 0 & t \end{bmatrix}$ , calculate the matrix function  $e^{A(t)}$ .