

國立臺北科技大學九十九學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the general solution for y .

(a) $y' = -y + xy^2$ (10%)

(b) $x^2 y'' - 2xy' + 2y = x(\ln x)$ (15%) (Integral formula: $\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$)

2. Solve the following the integro-differential equation for $y(t)$ using Laplace transform.

$ty'' + (1-2t)y' - y + e^t \int_0^t e^{-\tau} y(\tau) d\tau = 0, \quad y(0) = 2$ (20%)

3. Consider the line integral $\int_C \vec{F} \cdot d\vec{r}$, where \vec{r} is the position vector,

$\vec{F} = -3y^3 \vec{i} + (3x^3 + \cos y) \vec{j}$, $C: x^2 + y^2 = 4, z = 0$ (clockwise).

(a) Is the line integral independent of path? Why? (5%)

(b) Evaluate the line integral. (10%)

4. (a) Find the Fourier series of the function $f(x) = \pi - |x|$ for $-\pi \leq x \leq \pi$. (15%)

(b) Use the answer of (a) to find the sum of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$ (5%)

5. Solve the following partial differential equation:

$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ for $0 \leq x \leq 4, t \geq 0$

with boundary conditions: $y(0, t) = y(4, t) = 1$ for $t > 0$

and initial condition: $y(x, 0) = 2$ for $0 \leq x \leq 4$ (20%)