

# 國立臺北科技大學九十九學年度碩士班招生考試

系所組別：1511、1512 自動化科技研究所甲組

## 第一節 工程數學 試題

第一頁 共一頁

### 注意事項：

1. 本試題共 4 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (30%) A mass-spring-damper system is shown in Figure 1(a), where  $m$ ,  $c$ ,  $k$  are the mass, spring constant and damping coefficient, respectively. Then, the governing differential equation is

$$my'' + cy' + ky = f(t)$$

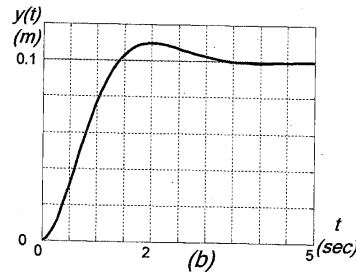
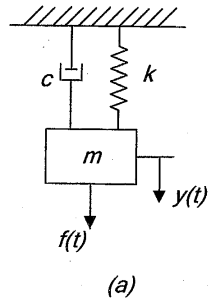


Figure 1

- (1) (15%) Considering a periodic driving force, say  $f(t) = A \cos(\omega t)$ , please determine the solution of  $y(t)$ .
  - (2) (8%) A force 2 Newton is applied to this system, the mass displacement of  $y(t)$  is plotted in Figure 1(b). Please determine the parameters  $k$ .
  - (3) (7%) If  $m = 5.2$  kg,  $c = 12$  Newton/m/sec, and input force  $f(t) = 2 \sin(t)$ , find the steady state response  $y_{ss}(t)$ .
2. (20%) Find the general solution of

$$y^{(4)} + 11y^{(3)} + 36y'' + 16y' - 64y = -3e^{-4x} + 2\cos(2x)$$

3. (20%) Find the eigenvalues and the corresponding eigenspaces.

(1)(10%)

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$$

(2)(10%)

$$A = \begin{bmatrix} -1 & -4 \\ 3 & -2 \end{bmatrix}$$

4. (30%) Let  $S = \text{Span} \left[ \begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ 2 \\ 2 \end{pmatrix} \right]$  be a subspace of

$R^4$ , and let  $b = (4, -1, 5, 1)^T$ .

(1)(10%) Find an orthonormal basis for  $S$ .

(2)(10%) Use your answer in (1) to find the projection  $p$  of  $b$  onto  $S$ .

(3)(10%) Given  $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & 5 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ . Use your answer in (2) to solve the least squares

problem  $Ax = b$ .