

國立臺北科技大學九十九學年度碩士班招生考試

系所組別：2401、2402 光電工程系碩士班

第一節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共九題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、 [10%]

Given that  $x$  and  $xe^x$  are solutions of the homogeneous equation corresponding to  $x^2y'' - x(x+2)y' + (x+2)y = 2x^3$ ,  $x > 0$ . Find the general solution by using the method of variation parameter.

二、 [15%]

Consider the following Sturm-Liouville problem

$$y'' - 2y' + (\lambda - 1)y = 0, y(0) = y(1) = 0$$

(a) Find the eigenvalues and eigenfunctions of the problem. [10%]

(b) Will the eigenvalues and eigenfunctions changed if the above boundary conditions are changed to be  $y(0) = y(5) = 0$ ? [5%]

三、 [15%]

Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2 - 1}$ .

四、 [10%]

Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - u = 2\sin x$ ,  $0 < x < \pi, 0 < y < \pi$ , with

$$\begin{cases} u(0, y) = 0, u(\pi, y) = 0, & 0 < y < \pi \\ u(x, 0) = 0, u(x, \pi) = 0, & 0 < x < \pi \end{cases}$$

五、 [10%]

Evaluate  $\oint_c (x^5 + 3y)dx + (5x - e^{y^3})dy$ , where  $c$  is the circle  $(x-1)^2 + (y-5)^2 = 4$ . The curve is oriented counterclockwise.

六、 [10%]

$$\text{Matrix } A = \begin{bmatrix} 3 & -4 \\ -4 & -3 \end{bmatrix}$$

Express the matrix function in a  $2 \times 2$  matrix form:  $\begin{bmatrix} f(x) & g(x) \\ g(x) & h(x) \end{bmatrix}$

七、 [10%]

Assume  $k > k' > 0$ , evaluate  $\int_0^{\infty} \frac{\sin kx \cdot \sin k'x}{x^2 + a^2} dx$ .

八、 [10%]

Prove  $\frac{1}{2\pi i} \oint_c e^{z+\frac{1}{z}} dz = \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}$ , where  $c: |z|=1$ .

九、 [10%]

Evaluate  $\int_0^{\infty} \frac{\cos(ax) - \cos(bx)}{x^2} dx$ , ( $a \geq 0, b \geq 0$ ).