

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：1310 1320 1330 車輛工程系碩士班甲乙丙組

第二節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共八題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. Find the general solution in each of the following differential equations.

(a)  $y' = xy^2 + y^2$ . (10%)

(b)  $x^2y'' - 3xy' + 3y = x$ . (15%)

二. Solve the initial value problem

$$y' + y = f(t), y(0) = 0, f(t) = \begin{cases} 1, & t < 2 \\ 0, & t \geq 2 \end{cases}. \quad (15\%)$$

三. Find the inverse Laplace transform of the function  $\frac{se^{-2s}}{(s^2+1)^2}$ . (10%)

四. Every vector in the set S is a linear combination of two vectors, (1,1,0) and (1,0,1), and every vector in the set K is a linear combination of two vectors, (1,2,-1) and (1,-1,2). Determine whether S=K and explain your reason. (10%)

五. Find the condition when the system  $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$  has a unique solution for the unknowns  $x$  and  $y$ . (10%)

六. Given a 4x4 matrix A and the determinant of A,  $|A|=3$ , find

(a)  $|A^{-1}|$ , (b)  $|2A|$ . (10%)

七. Suppose  $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = P \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$  transforms  $3x_1^2 + 2x_1x_2 + 3x_2^2 = 1$  to  $ay_1^2 + by_2^2 = 1$ .

Find the matrix P and two numbers,  $a$  and  $b$ . (15%)

八. Prove that if  $\lambda$  is an eigenvalue of a matrix M, then  $\alpha + \lambda$  is an eigenvalue of the matrix  $\alpha I + M$ , in which I is an identity matrix. (5%)