

# 國立臺北科技大學九十八學年度碩士班招生考試

系所組別：2220 電腦與通訊研究所乙組

## 第一節 工程數學 試題

第一頁 共一頁

### 注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、(11%) Consider a 8-ary symbol represented by three bits  $b_1 b_2 b_3$ . These three bits are transmitted through a digital communication system. Let

- event  $A_1$  = receiver make a decision error of  $b_1$ ,
- event  $A_2$  = receiver make a decision error of  $b_2$ ,
- event  $A_3$  = receiver make a decision error of  $b_3$ .

$P(A_1)=0.1$ ,  $P(A_2)=0.2$ ,  $P(A_3)=0.3$ , and events  $A_1$ ,  $A_2$ , and  $A_3$  are independent. The 8-ary symbol is correctly received, if all three bits are correctly received. Find the probability that the 8-ary symbol is correctly received.

二、(16%) Let  $A$  and  $B$  be two events, and  $P(A)=0.1$  and  $P(B)=0.3$ . (每小題 8 分)

1. If  $A$  and  $B$  are independent events, find  $P(A \cup B)$ .
2. If  $A$  and  $B$  are mutually exclusive (i.e.,  $A \cap B = \phi$ ), find  $P(A \cup B)$ .

三、(15%) Suppose that 5% of all transistors produced by factory A are defective, and 10% of all transistors produced by factory B are defective. An engineer first chose a factory and then obtained 100 transistors from that factory. If the probability that the engineer chose factory A is 0.3, and the probability that the engineer chose factory B is 0.7, find the expected number of defective transistors.

四、(15%) Normal cumulative distribution function  $\Phi(z)$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-t^2/2} dt$$

is the cumulative distribution function (CDF) of a Gaussian (normal) random variable with mean 0 and standard deviation 1. Let  $X$  be a Gaussian random variable with mean  $-3$  and variance 2. Express the probability  $P(X > 0)$  by using  $\Phi(z)$ .

五、(12%) Let  $\vec{u} = (2, 1, -3)$  and  $\vec{a} = (4, -1, 2)$ . Find the vector component of  $\vec{u}$  along  $\vec{a}$  and the vector component of  $\vec{u}$  orthogonal to  $\vec{a}$ .

六、(15%) Consider two vectors  $\vec{u}$  and  $\vec{v}$ . Let

$$\vec{a} = \vec{u} - \left( \vec{u} \cdot \frac{\vec{v}}{\|\vec{v}\|} \right) \frac{\vec{v}}{\|\vec{v}\|}.$$

If  $\vec{a}$  is not a zero vector, prove that  $\vec{v}$  and  $\vec{a}$  are orthogonal.

七、(16%) In the following questions, determine whether  $A$  is diagonalizable. If so, find a matrix  $P$  that diagonalizes  $A$ , and determine  $P^{-1}AP$ . (每小題 8 分)

1.  $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

2.  $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$