

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：2210 電腦與通訊研究所甲組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

True and False: Suppose there are three $N \times N$ matrices **A**, **B**, and **C**. Show whether the following statements are true or false.

1. If $AC = BC$, then $A = B$. (5%)

2. If $A^2 = A$, then $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (5%)

3. If both **A** and **B** are non-singular matrices, then $A + B$ is also non-singular. (5%)

4. If both **A** and **B** are non-singular matrices, then AB is also non-singular. (5%)

5. If AB is an invertible matrix, then both **A** and **B** are invertible. (5%)

二、

Find the eigenvalues of matrix $A = \begin{bmatrix} 4 & 3 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 \\ 1 & 7 & 10 & 1 & -7 \\ 2 & -1 & 0 & 5 & 0 \\ -3 & 1 & 6 & -4 & -3 \end{bmatrix}$. (10%)

三、

Find A^4 , where **A** is a matrix whose eigenvalues are 1 and 2 and whose eigenvectors are $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, respectively. (10%)

四、

Let random variables $Y = X^2$. Find the probability density function and expectation of Y for the following cases.

1. X takes the values -2, -1, 0, 1, 2, 3 with equal probability 1/6. (10%)

2. X is uniformly distributed in the interval (0,1). (10%)

五、

There are two random variables $X \in \{0, 1, 2\}$ and $Y = \begin{cases} 1, & \text{if } X = 0 \\ 0, & \text{if } X = 1, 2 \end{cases}$.

If $P(X=0) = P(X=1) = P(X=2) = 1/3$, and $P(Y=0) = P(Y=1) = 1/2$, then are X and Y orthogonal? uncorrelated? independent? (12%)

六、

Let $x(t)$ be a signal defined by $x(t) = A \cos(\omega t + \theta)$, where A is a Gaussian random variable with zero mean and unit variance, θ is a uniform random variable in the interval $(0, 2\pi)$, and ω is a positive constant. Suppose that A and θ are independent. Find the mean and variance of $x(t)$. (10%)

七、

If the probability of obtaining a "Head" in a toss experiment is considered as a random variable X , we may assume the probability density function of X is uniform in the interval (0,1). Now suppose that we have performed an experiment of tossing a coin 3 times, and 2 "Heads" are observed. What is the conditional density function of X , given this observation? (13%)

201-1-1