

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：2140 2150 電機工程系碩士班丁戊組

第二節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Given a probability space $(\Omega, \mathfrak{F}, P)$, the events A, B , and C form a partition of the sample space Ω . Events D and E are two disjoint events in the event space. The joint probabilities of these events are shown in the following table.

Events	A	B	C
D	$P(A \cap D)$ =0.0916	$P(B \cap D)$ =0.1988	$P(C \cap D)$ =0.0585
E	$P(A \cap E)$ =0.2144	$P(B \cap E)$ =0.2203	$P(C \cap E)$ =0.2164

- (a) (8%) Find $P(D)$ and $P(B/D)$.
 - (b) (4%) Could it be possible to find $P(A)$ by using the table above? Explain your answer. If the answer is "Yes", compute $P(A)$.
 - (c) (3%) Give the definition that a collection of subsets in Ω is a partition of the sample space Ω .
 - (d) (3%) Do events D and E form a partition of the sample space Ω ? (Explain your answer.)
2. X and Y are two random variables with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{3} & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ \frac{2}{3} & -1 \leq x < 0, -1 \leq y < 0 \\ 0 & \text{else} \end{cases}$$

- (a) (5%) Find the marginal probability density function $f_X(x)$.
 - (b) (5%) Find the conditional probability density function of Y given $X = x$.
 - (c) (5%) Find $P(X > Y)$.
3. Let X_1, X_2, X_3 , and X_4 be independent random variables with common probability mass functions $P(X_i = 1) = 1/3$ and $P(X_i = 0) = 2/3$ for $i = 1, 2, 3, 4$. Define the random variables Y_1, Y_2 , and Y_3 as $Y_i = \begin{cases} 1 & X_i \neq X_{i+1} \\ 0 & X_i = X_{i+1} \end{cases}$.
- (a) (6%) Find the probability mass function and cumulative distribution function of Y_1 .
 - (b) (4%) Find $E(Y_1 Y_2)$.
 - (c) (4%) Find $E(Y_1 Y_3)$.
 - (d) (3%) Are Y_1 and Y_3 independent? (Explain your answer.)

4. (15%) Let $A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. Find the inverse of and the determinant of A .

5. Let $B = \begin{bmatrix} -1 & 2 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ -6 & 3 & 6 & 0 & 0 \\ 0 & 0 & 0 & -12 & 25 \\ 0 & 0 & 0 & -9 & 18 \end{bmatrix}$.

- (a) (5%) Find the inverse of B ;
 - (b) (15%) Find all eigenvalues of B and the corresponding eigenvectors.
6. (15%) Prove in English that "If A is an $m \times n$ matrix, the dimension of the row space of A equals the dimension of the column space of A ". (No credit will be given if the answer is given in Chinese)