國立臺北科技大學九十八學年度碩士班招生考試

系所組別：2320 資訊工程系碩士班乙組

第一節　工程數學　試題

注意事項：

1. 本試題共 7 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) Let $A=\begin{bmatrix}1 & 0\\ 2 & 3\end{bmatrix}$.
   (a) Evaluate A^2. (5%)
   (b) Find the expression for A^k by diagonalizing A. (10%)

2. (10%) Suppose that T is a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$, T: $\mathbf{R}^2 \to \mathbf{R}^2$ with
   $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=\begin{bmatrix}3\\-2\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}1\\4\end{bmatrix}$.
   (a) Evaluate $T\left(\begin{bmatrix}3\\1\end{bmatrix}\right)$. (5%)
   (b) Find the standard matrix for T. (5%)

3. (15%) Let $A=\begin{bmatrix}-1 & 2\\ 2 & -3\\ -1 & 3\end{bmatrix}$ and $\mathbf{y}=\begin{bmatrix}4\\1\\2\end{bmatrix}$.
   (a) Suppose $\mathbf{z}$ is a vector in Col A that is closest to $\mathbf{y}$. Find $\mathbf{z}$. (Col A is the column space of A.) (10%)
   (b) Find an orthogonal basis for Col A. (5%)

4. (10%) Let A be an $m\times n$ matrix. Prove that rank(A^TA) = rank(A).

5. (15%) A box contains the numbers: 2, 3, 7, 8, 12, 15, 17, 21 and 28. Six numbers are picked randomly without replacement. What is the probability that the third largest number is 15?

6. (20%) Given the joint probability density function of the two-dimensional random variables X and Y is $f_{X,Y}(x,y)=\begin{cases}6e^{-(2x+3y)} & x\ge 0, y\ge 0\\ 0 & otherwise\end{cases}$ (每小題 5 分)
   Then compute (a) $P\{X>Y\}=?$ (b) $P\{X+Y\le 1\}=?$
   (c) $P\{\min(X,Y)\ge 1\}=?$ (d) $P\{\max(X,Y)\le 1\}=?$

7. (15%) Let $X_1,\cdots,X_n$ be independent and identically distributed random variables. Then find $E[X_1 \mid X_1+\cdots+X_n=x]=?$