

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the unit normal vector and the equation of the tangent plane to

the graph of  $z = \cos(2x + y)$  at the point  $(\frac{\pi}{2}, \frac{\pi}{4}, -\frac{1}{\sqrt{2}})$  (20%)

2. To evaluate the Laplace transform:  $\mathcal{L}\{t \int_0^t \sin \tau \, d\tau\}$  (10%)

3. Find the solution  $\frac{dy}{dt} + y = f(t)$ ,  $y(0) = 5$ , where

$f(t) = 0$ ,  $0 \leq t \leq \pi$ , and  $f(t) = 3 \cos t$ ,  $t \geq \pi$ . (15%)

4. Solve the equation:  $xy'' = y' + (y')^3$  (15%)

5. Transform the following partial differential equation into a standard

heat equation by  $u(x, t) = e^{\alpha x + \beta t} v(x, t)$  and solve it:

$$\frac{\partial u}{\partial t} = \left( \frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial u}{\partial x} \right), \quad 0 < x < 4, \quad t > 0$$

$$u(0, t) = u(4, t) = 0, \text{ for } t \geq 0, \quad u(x, 0) = 1 \text{ (25\%)}$$

6 Find the Fourier series of the function

$$f(x) = |x|, \quad -5 \leq x \leq 5 \text{ (15\%)}$$