

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：3110 3120 3150 土木與防災研究所甲乙戊組

第二節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Find a general solution $y(x)$ of the following differential equations.

1. $y'''' - 2y''' + y'' = 0$; (10分)
2. $y'' + 2y' + 5y = 16e^x + \sin 2x$; (15分)

二、Solve the following system of differential equations.

1. $x' = -2x + y$
 $y' = -4x + 3y + e^{-t}$, where $x' = dx/dt$ and $y' = dy/dt$. (10分)
2. $x'' = -5x + 2y$
 $y'' = 2x - 2y$, where $x'' = d^2x/dt^2$ and $y'' = d^2y/dt^2$. (15分)

三、Are the given vectors linearly independent or dependent? Why?

1. $[1 \ 2 \ 3], [4 \ 5 \ 6], [7 \ 8 \ 9]$ (5分)
2. $[1 \ 1 \ 1], [1 \ -1 \ 1], [1 \ 1 \ -1]$ (5分)

四、Find the eigenvalues and eigenvectors of the following matrices.

1. $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ (10分)

2. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, (eigenvalues are known as 5, -3, -3) (10分)

五、Are the following matrices symmetric? Skew-symmetric? Orthogonal? Why?

1. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (5分)
2. $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ (5分)

六、Find out what type of conic section the following quadratic form represents.

Transform it to principal axes. Express $[x_1 \ x_2]^T$ in terms of the new coordinate vector $[y_1 \ y_2]^T$. (10分)

$$Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$$