

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：1201 1202 1203 製造科技研究所不分組

第一節 微分方程 試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve $\frac{dy}{dx} = \frac{1}{x-y+1}$, $y(1) = 0$. (20 分)
2. If the nonexact differential equation, $M(x, y)dx + N(x, y)dy = 0$, has integrating factors of the form $\phi(x, y) = f(xy)$, derive the $\phi(x, y)$ in terms of M, N, x, y . (20 分)
3. Solve $4x^2 y'' + y = \sqrt{x}$. (20 分)
4. Consider the initial value problem $y'' + 3y' + 2y = B\delta(t)$, $y(0) = 1, y'(0) = 0$, where B and $\delta(t)$ are an arbitrary constant and the Dirac delta function, respectively.
 - (1) Find the solution $y(t)$. (8 分)
 - (2) What are the initial values $y(0)$ and $y'(0)$ derived from $y(t)$? (4 分)
 - (3) Using the information from question (2), what physical phenomenon does the Dirac delta function model? (8 分)
5. Solve the system

$$\mathbf{X}' = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \mathbf{X}$$

with given initial conditions

$$\mathbf{X}(0) = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}. \quad (20 \text{ 分})$$