國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2230 電腦與通訊研究所丙組

第二節　電磁學　試題

填准考證號碼

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、For two quarter circular line charges of density $2\rho_\ell$ and $-\rho_\ell$, respectively, located on the *x*-*y* plane, as shown in Fig. 1 below, determine the following quantities at any point $(0, 0, z)$ on the *z*-axis,

1. the electric potential V, (5%)
2. the electric field intensity $\mathbf{E}$. (10%)

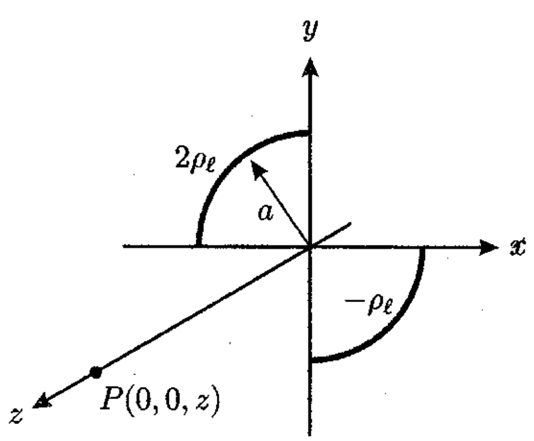

Figure 1.

二、An element shown in Fig. 2 below is defined by the following surfaces:

- $r = r_1$ and $r = r_2$,
- $\theta = \frac{\pi}{2}$ and $\theta = \beta$,
- $\phi = 0$ and $\phi = \alpha$.

Compute the following if the material of the element is characterized by a conductivity of σ:

1. the resistance of this element if the surface at $r = r_1$ has $V = 0$ and the surface at $r = r_2$ has $V = V_0$. Then, determine the capacitance of this element if the material is characterized by a permeability of ϵ. Neglect fringing. (8%)
2. the resistance of this element if the surface at $\theta = \pi/2$ has $V = 0$ and the surface at $\theta = \beta$ has $V = V_0$. Neglect fringing. (6%)
3. the resistance of this element if the surface at $\phi = 0$ has $V = 0$ and the surface at $\phi = \alpha$ has $V = V_0$. Neglect fringing. (6%)

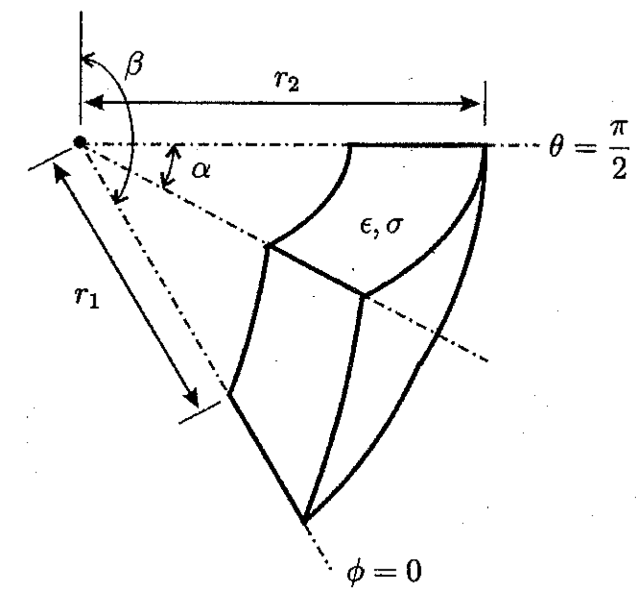

Figure 2.

注意：背面尚有試題

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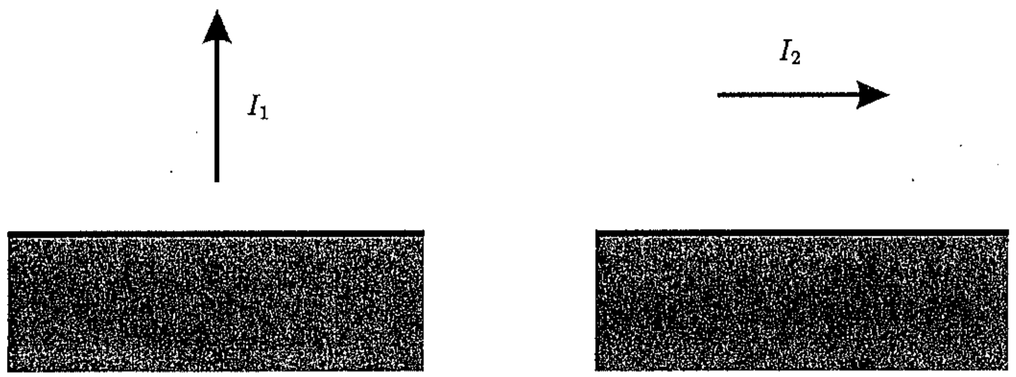
三、 Conducting wires above a conducting plane carry currents I_1 and I_2 in the directions shown in Fig. 3 below. Keeping in mind that the direction of a current is defined in terms of the movement of positive charges, what are the directions of the image currents corresponding to I_1 and I_2? (15%)



(a) (b)

Figures 3.

四、 The electromagnetic fields in a rectangular waveguide shown in Fig. 4 below are given by

$$\mathbf{E} = C\frac{\omega\mu_o a}{\pi}\sin\left(\frac{\pi x}{a}\right)\sin(\omega t-\beta z)\hat{y}$$

$$\mathbf{H} = -C\frac{\beta a}{\pi}\sin\left(\frac{\pi x}{a}\right)\sin(\omega t-\beta z)\hat{x}+C\cos\left(\frac{\pi x}{a}\right)\cos(\omega t-\beta z)\hat{z}$$

where C is a constant and $\omega = 2\pi f$, with f the frequency of excitation. The walls of the waveguide are assumed to be perfect conductors. Determine the surface charge densities and surface current densities on those walls. (20%)

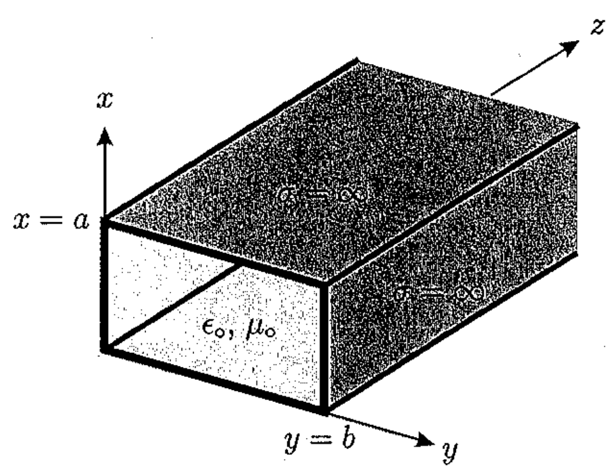


Figure 4.

五、 Determine the polarization of the following uniform plane waves:

1. $\mathbf{E} = 1\cos(\omega t+\beta z)\hat{x}+2\cos(\omega t+\beta z)\hat{y}$, (2%)
2. $\mathbf{E} = 2\cos(\omega t+\beta z)\hat{y}$, (2%)
3. $\mathbf{E} = 1\cos(\omega t-\beta z)\hat{x}-1\sin(\omega t-\beta z)\hat{y}$, (2%)
4. $\mathbf{E} = 1\cos(\omega t+\beta z)\hat{x}-2\sin(\omega t+\beta z-45°)\hat{y}$. (2%)
5. $\mathbf{E} = 1\cos(\omega t+\beta z)\hat{x}+1\sin(\omega t+\beta z)\hat{y}$, (2%)

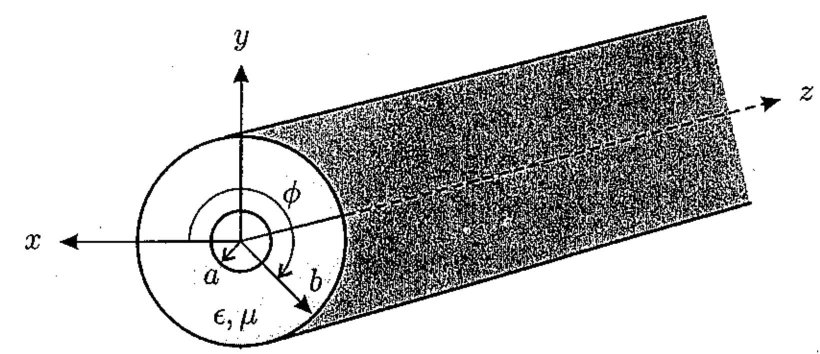
(10%)

六、 A *TEM* wave propagates within a coaxial structure shown in Fig. 5 below with the fields given by

$$\mathbf{E} = \hat{\rho}E_\rho = \hat{\rho}\frac{E_0}{\rho}e^{-j\beta z}$$

$$\mathbf{H} = \hat{\phi}H_\phi = \hat{\phi}\frac{H_0}{\rho}e^{-j\beta z}.$$

Perfect conductors and lossless medium in between are assumed. Derive in detail the time-harmonic transmission-line equations for V and I and the expressions for the associated line capacitance C and line inductance L of this coaxial structure through the two *Curl* equations of the time-harmonic Maxwell's equations. (20%)



Figures 5.

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Laplace's equation in cylindrical coordinates:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

and in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0.$$

Some useful integrals:

$$\int \sin x dx = -\cos x + C, \qquad \int \cos x dx = \sin x + C,$$

$$\int \tan x dx = -\ln(\cos x) + C, \qquad \int \cot x dx = \ln(\sin x) + C,$$

$$\int \sec x dx = \ln\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] + C, \qquad \int \csc x dx = \ln\left(\tan\frac{x}{2}\right) + C.$$

The *Curl* operation in orthogonal coordinates is given below

$$\nabla \times \mathbf{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} \hat{u}_1 h_1 & \hat{u}_2 h_2 & \hat{u}_3 h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix},$$

where h_i (i=1, 2, or 3) is the metric coefficient and u_i (i=1, 2, or 3) is the coordinate.

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