

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2140 2150 電機工程系碩士班丁戊組

第二節 工程數學 試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Let $P_n(R)$ consist of all real-coefficient polynomials having degree less than or equal to n . Define $T: P_2(R) \rightarrow P_3(R)$ by

$$T(f)(x) = \int_0^x f(t) dt.$$

Let the ordered bases of $P_2(R)$ and $P_3(R)$ be $\alpha = \{1, x, x^2\}$ and $\beta = \{1, (1-x), (1-x)^2, (1-x)^3\}$.

1. (10%) Find the matrix representation of $[T]_{\beta}^{\alpha}$.
2. (10%) Consider the polynomial $p(x) = 1 - 2x + 6x^2$. Find the coordinate vector of $T(p)$ relative to β .

二、Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

1. (10%) Find $A^{1/2}$.
2. (10%) Find the matrix $C \in R^{3 \times 2}$ such that $A = CC^T$.

三、(10%) Let U and V be vector spaces, and let $L: U \rightarrow V$ be a linear transformation.

Show that L is one-to-one (injective) if and only if the null space of L is $\{0\}$.

四、Random variable X has PMF

$$P_X(x) = \begin{cases} c(1/2)^x & x = 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

1. (10%) What is the value of the constant c ?
2. (10%) What is $P[X \leq 2]$?

五、Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} (x+y)/8 & 0 \leq x \leq 2; 0 \leq y \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

Let $A = \{X \leq 1\}$.

1. (10%) What is $P[A]$?
2. (10%) Find $f_{X,Y|A}(x,y)$.
3. (10%) Find $f_{Y|A}(y)$.