

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2111 2112 2120 2130 電機工程系碩士班甲乙丙組

第二節 工程數學 試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

- 一、Let $P_n(\mathbb{R})$ consist of all real-coefficient polynomials having degree less than or equal to n . Define $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ by

$$T(f)(x) = \int_0^x f(t) dt.$$

Let the ordered bases of $P_2(\mathbb{R})$ and $P_3(\mathbb{R})$ be $\alpha = \{1, x, x^2\}$ and $\beta = \{1, (1-x), (1-x)^2, (1-x)^3\}$.

1. (10%) Find the matrix representation of $[T]_{\beta}^{\alpha}$.
2. (10%) Consider the polynomial $p(x) = 1 - 2x + 6x^2$. Find the coordinate vector of $T(p)$ relative to β .

二、Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

1. (10%) Find $A^{1/2}$.
 2. (10%) Find the matrix $C \in \mathbb{R}^{3 \times 2}$ such that $A = CC^T$.
- 三、(10%) Let U and V be vector spaces, and let $L: U \rightarrow V$ be a linear transformation. Show that L is one-to-one (injective) if and only if the null space of L is $\{0\}$.

- 四、Consider the following differential equation

$$x^3 y''' + 9x^2 y'' + 19xy' + 8y = 0.$$

1. (10%) Find the basis of the solution space of the differential equation.
2. (10%) Show the elements of the basis are indeed linearly independent.

- 五、(15%) Find the general solution of the following differential equation $(y - xy^4)dx + xdy = 0$.

- 六、Consider the function shown in Fig. 1

1. (5%) Obtain the corresponding mathematical expression.
2. (10%) Find the Laplace transform of the function.

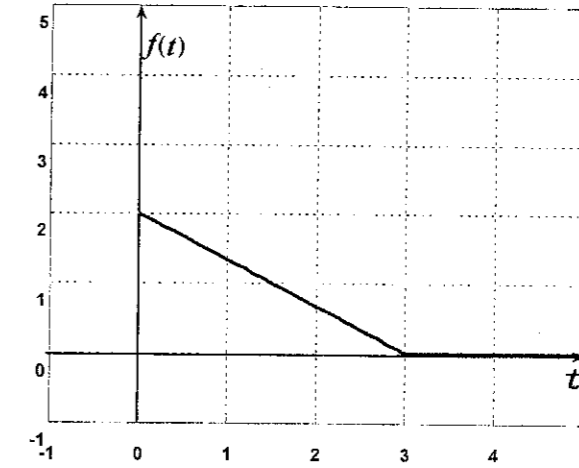


Fig. 1