國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2401　2402 光電工程系碩士班不分組

第一節　工程數學　試題

填准考證號碼

第一頁　共一頁

注意事項：
1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1.(10%)

Using the relation $\nabla\cdot(f\vec{A})=f(\nabla\cdot\vec{A})+\vec{A}\cdot(\nabla f)$ to evaluate the volume integral

$$J=\iiint(r^3+1)\nabla\cdot\left(\frac{\hat{r}}{r^2}\right)dv$$

For a sphere with radius R center at the origin (10%)

2.(15%)

Solve the following diffraction equation

(a) $y''+4y'+4y=3xe^{-2x}$ (5%)

(b) $x^2y''-4xy'+6y=x^4\sin x$ (10%)

3.(20%)

Using the Laplace transformation to

(a) Calculate that (8%)

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$$

(b) find $N_1(t)$, $N_2(t)$, $N_3(t)$ from the following equation (12%)

$$\frac{dN_1}{dt}=-\lambda_1N_1,$$

$$\frac{dN_2}{dt}=\lambda_1N_1-\lambda_2N_2$$

$$\frac{dN_3}{dt}=\lambda_2N_2$$

Here, $N_1(0)=N_0$, $N_2(0)=0$, $N_3(0)=0$

4.(20%)

(a) Expand Dirac delta function $\delta(x-t)$ in a Fourier series. (10%)

(b) Use the result of (a) to verify that (10%)

$$\delta(\varphi_1-\varphi_2)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{-im(\varphi_1-\varphi_2)}$$

5.(13%)

(a) Planck's theory of quantized oscillators led to an average energy

$$\langle\varepsilon\rangle=\frac{\sum_{n=1}^{\infty}n\varepsilon_0\exp(-n\varepsilon_0/kT)}{\sum_{n=0}^{\infty}\exp(-n\varepsilon_0/kT)}$$

Where ε_0 was a fixed energy. Identify numerator and denominator as binominal expansions and show that the ratio is (10%)

$$\langle\varepsilon\rangle=\frac{\varepsilon_0}{\exp(\varepsilon_0/kT)-1}$$

(Hint, binomial expansion is $(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\frac{m(m-1)(m-2)}{3!}x^3+\ldots\ldots$)

(b)Show that $\langle\varepsilon\rangle$ of part (a) reduce to kT, the classical result, for $kT>>\varepsilon_0$. (3%)

6.(22%)

Use residues theorem to evaluate the following integrations

(a) $\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dx$ (10%)

(b) $\int_{0}^{\infty}\frac{1}{1+x^3}dx$ (12%)

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