國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1201、1202、1203 製造科技研究所

第一節 微分方程 試題

注意事項：
1. 本試題共4大題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、 Consider the differential equation

$$M(x,y)dx + N(x,y)dy = 0 \tag{1}$$

where M and N have continuous first partial derivatives at all points (x,y) in a rectangular domain D.

1. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact in D, then

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x} \tag{2}$$

for all $(x,y) \in D$. Please prove the result. (5分)

2. Conversely, if

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x} \tag{3}$$

for all $(x,y) \in D$. Then the differential equation (1) is exact in D. Please prove the result. (10分)

Hint:

This must prove that there exists a function F such that

$$\frac{\partial F(x,y)}{\partial x} = M(x,y) \tag{4}$$
$$\frac{\partial F(x,y)}{\partial y} = N(x,y) \tag{5}$$

for all $(x,y) \in D$.

3. Consider the differential equation

$$(3y + 4xy^2)dx + (2x + 3x^2y)dy = 0 \tag{6}$$

Please find general solution of the nonexact differential equation (6) using method of integrating factor. (10分)

二、 About the orthogonal and oblique trajectories:

1. Find the orthogonal trajectories of the family of parabolas $y = cx^2$. (10分)
2. Find a family of oblique trajectories that intersect the family of straight lines $y = cx$ at angle 45°. (15分)

Hint:
orthogonal(正交的); trajectories(曲線); intersect(相交)

$$tan(\alpha \pm \beta) = \frac{tan\alpha \pm tan\beta}{1 \mp tan\alpha\, tan\beta} \tag{7}$$
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctan\frac{x}{a} \tag{8}$$

三、 A 1kg weight is attached to the lower end of a coil spring suspended from the ceiling (acceleration of gravity g=10m/s). The weight comes to rest in its equilibrium position, thereby stretching the spring 62.5cm. The weight is then pulled down 50cm below its equilibrium position and released at t=0. No external forces are present; but the resistance of the medium in kilograms is numerically equal to $4(\frac{dx}{dt})$, where $\frac{dx}{dt}$ is the instantaneous velocity in meter per second.

1. Set up the differential equation for the motion and list the initial conditions. (10分)

Hint:
suspended(懸掛的); ceiling(天花板); stretching(延伸)
Hooke's Law: $F = ks$, This is a question of free damped motion.

2. Solve the initial-value set up in part 1. to determine the displacement of the weight as a function of the time. (15分)

四、 Consider the initial-value problem

$$\frac{dy}{dx} = 2x + y \tag{9}$$
$$y(0) = 1 \tag{10}$$

1. Apply the Runge-Kutta method to approximate the values of the solution y at x=0.2 and x=0.4 using h=0.2. Carry the intermediate calculations in each step to five figures after the decimal point, and round off the final results of each step to four such places. (10分)

Hint:

$$x_{n+1} = x_n + h, \tag{11}$$
$$k_1 = hf(x_n, y_n), \tag{12}$$
$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}), \tag{13}$$
$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}), \tag{14}$$
$$k_4 = hf(x_n + \frac{h}{2}, y_n + \frac{k_3}{2}), \tag{15}$$
$$K_n = \frac{1}{6}(k_1 + k_2 + k_3 + k_4), \tag{16}$$
$$y_{n+1} = y_n + K_n. \tag{17}$$

注意：背面尚有試題

2. Find the exact solution of the problem and compare the results obtained in part 1. with the exact values. (15分)

Hint:

$$\int xe^{ax}dx = \frac{1}{a^2}(ax-1)e^{ax}+c \tag{18}$$

$$e^{0.2} = 1.2214028 \tag{19}$$

$$e^{0.4} = 1.4918247 \tag{20}$$