國立臺北科技大學九十六學年度碩士班招生考試

系所組別：4112 工業工程與管理系碩士班甲組

第二節 作業研究（選考）試題

注意事項：
1. 本試題共四題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，型式如 4. 說明，否則不予計分。
4. 請自行在答案卷首頁依序標註題號並畫出題目下方填答的表格（Answers of …），然後將答案填入。答案卷第二頁之後必須有所有答案的計算過程與推導。

1. Given the following LP problem:

$$\text{Max } Z = [3 \quad 2 \quad 3]X$$

s.t.

$$\begin{bmatrix} -2 & -1 & -1 \\ 3 & 4 & 2 \end{bmatrix} X \geq \begin{bmatrix} -2 \\ 8 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq 0$$

(1) Please apply Two-Phase method to find the optimal solution if it exists. If there is no optimal solution, please explain your opinions.

Answers of 1-(1):

Optimum Z*	Basic variable_1	Basic variable_2
4_points	4_points	4_points

(2) Use the information in (1) to identify the range for c_1 and c_2 (the first and second coefficients in the objective function) respectively such that the optimal basis still remains the same.

(3) Show your calculation to give the value of $\partial Z/\partial x_1$ in the final iteration.

(4) Please identify the pivot number (element) in the last iteration and explain why.

(5) Please identify the new optimum Z* if the right hand side is changed to $[-3, 7]^T$. DO NOT solve it from the beginning.

Answers of 1-(2)~(5):

1-(2) Range of c_1	1-(2) Range of c_2	1-(3) $\partial Z/\partial x_1$	1-(4) pivot number	1-(5) new optimum Z*
2_points	2_points	2_points	2_points	5_points

2. Consider the three-period inventory problem. At the beginning of each period, a firm must determine how many units should be produced during the current period. During a period, if x units are produced, a production cost $c(x)$ is incurred, where $C(0) = 0$, and for $x > 0$, $c(x) = 3+2x$. Production capacity during each period is limited to 4 units. The demand is random and observed after production occurs. Each period's demand is equally likely to be 1 or 2 units. After meeting the current period's demand out of current production and inventory, the firm's end-of-period inventory is evaluated, and a holding cost of $1 per unit is assessed. Because of limited space capacity, the inventory at the end of each period cannot exceed 3 units. It is required that all demand be met on time. Any inventory on hand at the end of period 3 can be sold at $2 per unit. At the beginning of period 1, the firm has 1 unit of inventory. Please apply dynamic programming to determine a production policy that minimizes the expected net total cost incurred during the three periods.

Answers of 2:

Amount produced in period 1	Amount produced in period 2	Total cost
8_points	8_points	8_points

3. Consider the transportation problem having the following cost parameter table

	City1	City2	City3	SUPPLY
Plant1	5	9	10	24
Plant2	11	6	8	19
Plant3	12	13	7	17
DEMAND	21	10	19	

(1). use Vogel's approximation method to find a basic feasible solution and its total cost.

(2). find the optimal solution and its total cost.

(3). consider the following demands of each city and formulate its Simplex tableau. DO NOT solve it.

	City1	City2	City3
Minimum demand	21	10	19
Maximum demand	28	13	∞

Answer of 3.

3-(1)-a: VAM BFS	3-(1)-b: Total cost	3-(2)-a: optimal solution	3-(2)-b: total cost	3-(3) Simplex tableau
In tabular format		In tabular format		In tabular format
10_points		10_points		6_points

Note: 其中 3-(1)-a、3-(2)-a、3-(3) 需另外畫出最後答案的表格，不需在上列表格內作答。

注意：背面尚有試題

4.

(1). In a Markov chain, given two states i and j, a path from i and j is a sequence of transitions that begins in i and ends in j, such that each transition in the sequence has a positive probability of occurring. A state is **reachable** from state i if there is a path leading from i to j. Two states i and j are said to __4-(1)-a__ if j is reachable from i, and i is reachable from j. A set of states S in a Markov chain is a __4-(1)-b__ set if no state outside of S is reachable from any state in S. A state i is a __4-(1)-c__ state if there exists a state that is reachable from i, but the state i is not reachable from state j. A state i is __4-(1)-d__ with period $k>1$ if k is the smallest number such that all paths leading from state i back to state j have a length that is a multiple of k. If all states in a chain are recurrent, aperiodic and communicate with each other, the chain is said to be __4-(1)-e__.
（在空格中填入正確英文名詞，每一空格 1 分）

(2). You are selling ice cream. Suppose that the sales of ice cream are related to the weather. Your will have a daily net loss of \$500 if it is a raining day. If not, your will have a daily net earning of \$500. During this season, the probability of a raining day is 1/3. At the beginning of April 1st, you have \$1000 capital. Your goal is to increase your capital to \$1500. As soon as you do, you stop selling. You also stop selling if your capital is reduced to \$0. What is the probability of your capital to be \$0, \$500, \$1000, and \$1500 at the end of April 4th ? **(8_points)**

(3). Consider the following transition matrix:

$$P=\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c}\begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0 & 0.7 \end{bmatrix}\end{array}$$

(a) compute the steady-state probabilities of the Markov chain **(6_points)**
(b) compute the expected(or mean) recurrence time for each state **(6_points)**

Answers of 4-(1)

4-(1)-a	4-(1)-b	4-(1)-c	4-(1)-d	4-(1)-e
1_point	1_point	1_point	1_point	1_point

Answers of 4-(2)

Probability of \$0	Probability of \$500	Probability of \$1000	Probability of \$1500
2_points	2_points	2_points	2_points

Answers of 4-(3)-a　　　Answers of 4-(3)-b

Steady-State Probability of state 0	Steady-State Probability of state 1	Steady-State Probability of state 2	Expected recurrence time of state 0	Expected recurrence time of state 1	Expected recurrence time of state 2
2_points	2_points	2_points	2_points	2_points	2_points