

國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1710、1720、1740 電腦與通訊研究所甲、乙、丁組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、A store sells three new brands of DVD drivers. Of its sales, 60% are brand Q, 30% and 10% are brand R and S, respectively. All manufactures offer a one-year warranty on parts and labor. The store found that 25% of brand Q's DVD drivers require warranty repair work, whereas 20% and 10% are for brands R and S, respectively.

- (a) Determine the probability that a randomly selected buyer has bought a brand R or S DVD driver that will need repair while under warranty. (6%)
- (b) Find the probability that a randomly selected buyer has a brand Q DVD driver that will not need repair in one-year warranty. (6%)

二、The distribution of the amount of new Core-2-Due PCs sold by a supply company in a given month is a continuous random variable X with pdf

$$f(x) = \begin{cases} \frac{3}{2}(x^2 - 1), & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the expected value $E[3X-1]$. (6%)
- (b) Calculate the variance of X , $V[X]$. (8%)

三、Suppose X and Y are continuous random variables, the joint pdf of X and Y , is given by

$$f(x, y) = \begin{cases} 6xy, & 0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq x+2y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Derive the marginal pdf of Y , $f_Y(y)$. (5%)
- (b) Find $P(0.5 \leq Y \leq 1)$. (5%)
- (c) Find $P(0 \leq X \leq 1, 0 \leq Y \leq 0.5)$. (5%)
- (d) Find $P(1 \leq X \leq 2)$. (5%)

四、Show that if any $(n \times n)$ matrices A and B are similar, then both A and B have the same characteristic polynomial. (12%)

五、Let the two square matrices P and Q be given by

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{bmatrix}, \quad Q = \begin{bmatrix} -a^3 & a & a^3 & -a^2 \\ 0 & b & 0 & 0 \\ c^3 & -c^2 & -c & 0 \\ d & d^3 & 0 & 0 \end{bmatrix}$$

- (a) Derive the determinant of P^T , $\det(P^T)$, in terms of a, b, c , and d . (6%)
- (b) Calculate $\det(Q^{-1}P^1Q^2)$ for $a=3, b=6, c=8$, and $d=9$. (6%)

六、Assume that the square matrix A is defined as follows:

$$A = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

- (a) Determine the eigenvalues and corresponding eigenvectors for A . (6%)
- (b) Calculate $[a, 2a] \cdot A^7$, $a \in R$ and $a \neq 0$. (6%)
- (c) Find $[2, 1] \cdot A^{10}$. (6%)

七、(a) Find a least-squares solution of the inconsistent system $Ax = b$, where (6%)

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 2 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

- (b) Determine the corresponding least-squares error. (6%)