

國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1640、1650 電機工程系碩士班丁、戊組

第二節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共 7 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Let  $T$  be the linear operator on  $\mathbb{R}^3$  defined by  $T(x,y,z) = (2y+z, x-4y, 3x)$ .
  - (a) (15%) Find the matrix of  $T$  in the basis  $\{f_1=(1,1,1), f_2=(1,1,0), f_3=(1,0,0)\}$ .
  - (b) (10%) Verify that  $[T]_f[v]_f = [T(v)]_f$  for any vector  $v \in \mathbb{R}^3$ .
2. (10%) For the following matrix  $A$ , find a nonsingular matrix  $P$  such that  $P^{-1}AP$  is diagonal.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 7 & -5 \\ 2 & -5 & 8 \end{bmatrix}$$

3. (15%) To find the  $\det(A)$  that

$$A = \begin{bmatrix} a+x_1 & a & a & \cdots & a \\ a & a+x_2 & a & \cdots & a \\ a & a & a+x_3 & \cdots & a \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a & a & a & \cdots & a+x_n \end{bmatrix}$$

4. (a) (9%) Two cards are drawn in succession, without replacement, from an ordinary deck of playing cards (The total number of cards is 52.). Find  $P(A)$ ,  $P(B)$  and  $P(A \cap B)$ , where  $A$  is the event that the first card is a black ace and  $B$  is the event that the second card is a 9 or 10.
- (b) (3%) Are the events  $A$  and  $B$  independent? Explain your answer.

5. (a) (6%) Let  $X$  be a random variable. State the basic three properties of the cdf  $F_X(x) \triangleq P(X \leq x)$ .
- (b) (8%) Express the joint cdf  $F_{ZW}(z,w)$  in terms of the joint pdf  $f_{XY}(x,y)$  where  $Z = \max(X,Y)$  and  $W = \min(X,Y)$ .
6.  $X$  is Gaussian random variable with pdf  $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$ .
  - (a) (4%) Find the moment generating function of  $X$ .
  - (b) (6%) Show that  $E[|X|^{k+2}] = (k+1)\sigma^2 E[|X|^k]$  for nonnegative integer  $k$ .
  - (c) (4%) Compute  $E|X|^k$  for every positive integer  $k$ .
7. (a) (4%) State the definition that two random variables  $X, Y$  are uncorrelated.
- (b) (6%) Give an example that two random variables  $X, Y$  are uncorrelated but dependent. Justify your answer.