

國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1810 資訊工程系碩士班甲組

第二節 離散數學與演算法 試題

第一頁 共二頁

注意事項：

1. 本試題共十一題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、(3 pts) Which one is false?

- (A) Every infinite subset of a countably infinite set is countable infinite.
- (B) $\{p \mid p \text{ is a prime number}\}$ is a countably infinite set.
- (C) $\{i+i^2+i^3 \mid i \in \mathbb{N}\}$ is a countably infinite set.
- (D) No infinite set can be put into one-to-one correspondence with at least one of its proper subsets.

二、(3 pts) Let x_n be a sequence satisfying $x_{n+1}=3x_n-2x_{n-1}$ with $x_0=2, x_1=3$, what is x_n ?

- (A) $n+2$
- (B) n^2+2
- (C) 2^n+1
- (D) 2^n-1

三、(4 pts) Let R and S be two relations. Please list the statements which are **TRUE** among the following statements. You must correctly list all the true statements to get the credits. There is no partial credit on this problem.

- (A) If R and S are transitive, the $R \cup S$ is transitive.
- (B) If R is transitive, the R^{-1} is transitive, where R^{-1} is the inverse of R .
- (C) If R and S are symmetric, then $R \cap S$ is symmetric
- (D) If R and S are anti-symmetric, then $R \cup S$ is anti-symmetric
- (E) If R is anti-symmetric, then R^{-1} is anti-symmetric
- (F) S is anti-symmetric if S is asymmetric and irreflexive.

四、(10 pts) Let R and S be equivalence relations on X .

- (A) Show that $R \cap S$ is an equivalence relation on X .
- (B) Describe the equivalence classes of $R \cap S$ in terms of the equivalence classes of R and the equivalence classes of S .

五、(10 pts) Please use induction to show

$$\cos x + \cos 2x + \dots + \cos nx = \frac{\cos[(x/2)(n+1)]\sin(nx/2)}{\sin(x/2)}$$

provided that $\sin(x/2) \neq 0$.

六、(10 pts) Let $(S, +, \cdot, ', 0, 1)$ be a Boolean algebra and let A be a subset of S . Show that $(A, +, \cdot, ', 0, 1)$ is a Boolean algebra if and only if $1 \in A$ and $x \cdot y' \in A$ for all $x, y \in A$.

七、(10 pts) Let $\Sigma = \{a, b\}$. Show that the following language M is not regular.

$$M = \{w \in \Sigma^* \mid w \text{ has equal number of } a\text{'s and } b\text{'s}\}$$

八、(10 pts) Given an array of N integers, write an algorithm to find the median of these integers as fast as possible. Analyze the running time of your algorithm on the average and in the worse case. Do you think there's more efficient algorithm than yours?

九、(10 pts) Match up each quantity with the best matching asymptotic value in the following:

- (A) 1
- (B) $\log N$
- (C) N
- (D) $N \log N$
- (E) N^2

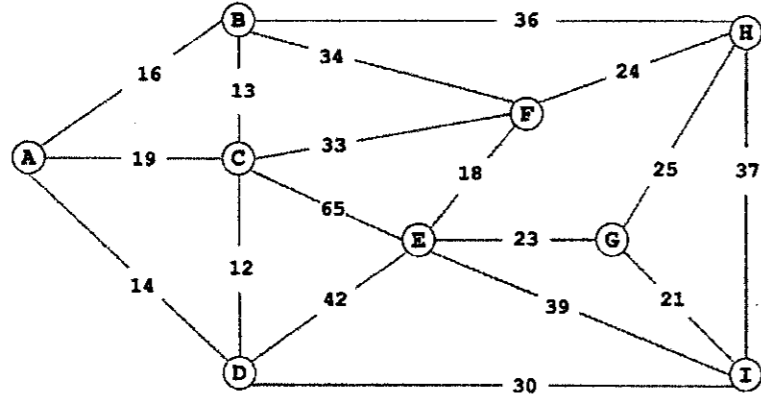
You may use a letter more than once.

1. Number of comparisons to sort a file which is already sorted with insertion sort.
2. Number of comparisons required to sort a file in reverse order with bubble sort.
3. Number of comparisons for selection sort of files with large records and small keys.
4. Maximum number of comparisons for sorting a file of N elements with quicksort in the worse case.
5. Number of comparisons for a search in a binary search tree with N random keys on the average.

注意：背面尚有試題

十、(15 pts) The *eight queens problem* is to put eight chess queens on an 8*8 chessboard such that no one can capture any others (that is, no two queens share the same row, column, or diagonal). Write an algorithm to find all possible solutions to the problem. Show the time complexity of your algorithm.

十一、(15 pts) Consider the following weighted graph G :



1. (5 pts) Please find a minimum spanning tree of G by Kruskal's algorithm. Please show your work step by step.
2. (10 pts) Please show that Kruskal's algorithm computes the minimum spanning tree of a graph in $O(|E| \log |E|)$ steps.