

國立臺北科技大學九十六學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共 5 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve  $2x^2y \frac{d^2y}{dx^2} + y^2 = x^2 \left(\frac{dy}{dx}\right)^2$  by using two consecutive substitutions.

$y = vx$  is suggested as the first substitution, and  $x = e^t$  as the second one.

(20%)

2. Find the  $y_1$  solution of the following system of ordinary differential

equations by using Laplace transform. (20%)

$$\frac{dy_1}{dt} = 2y_1 + y_2$$

$$\frac{dy_2}{dt} = 4y_1 + 2y_2 + 64tu(t-1), \text{ where } u(t-1)=0 \text{ as } t < 1$$

$$u(t-1)=1 \text{ as } t > 1$$

$$y_1(0)=2 \text{ and } y_2(0)=0.$$

3. Find the solution of the following ordinary differential equation by Frobenius method. (20%)

$$4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0.$$

4. C is a smooth wire with coordinate functions  $x=\sin(t)$ ,  $y=\cos(t)$ , and  $z=1$  for  $0 \leq t \leq \pi/2$ . The density function along the wire is  $\rho(x,y)=xy$  (g/cm).

Evaluate the mass of the wire. (20%)

5. Use polar coordinate to determine the volume of the solid that is under the hemisphere  $x^2 + y^2 + z^2 = 1$  and above the region R bounded by the

circle  $x^2 + y^2 = y$ . (Note: Use the symmetry of the circle to evaluate the volume of the solid) (20%)