國立臺北科技大學九十五學年度碩士班招生考試

系所組別：4111、4112、4121、4122 工業工程與管理系碩士班甲乙組

第一節 統計學 試題

填 准 考 證 號 碼

注意事項：

1. 本試題共 17 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

Part I. There are 15 questions in this part. Choose the best answer for each question. There are 4 points for each question totally 60 points.

Note: Normal Distribution Z.1=1.282, Z.05=1.645, Z.025=1.96, Z.01=2.326, Z.005=2.576, $\Phi(Z>1.5)=0.0668$, $\Phi(Z>1.4)=0.0808$, $\Phi(Z>1)=0.1587$, $\Phi(Z>2)=0.0228$

1. If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$. What is the variance of the random variable Z = -2X + 4Y – 3.
   a. 2
   b. 77
   c. 68
   d. 28

2. Suppose the President wants an estimate of the proportion of the population that support his current policy toward Mainland China. The President wants the sampling error to be within 4%. Assume a 95 percent level of confidence. The secretary estimated the proportion supporting current policy to be .60. How large a sample is required?
   a. 577
   b. 576
   c. 288
   d. 294

3. In question 2, how large would the sample have to be if the estimate by the secretary was not available?
   a. 307
   b. 601
   c. 600
   d. 306

4. The proportion of adults living in Taipei who are college graduates is estimated to p =0.3. To test this hypothesis, a random sample of 15 adults is selected. If the number of college graduates in our sample is anywhere from 2 to 7, we shall accept the null hypothesis that $p = 0.3$; otherwise, we shall conclude that $p \neq 0.3$. What is the type I error, α, assuming $p = 0.3$?

$\sum\limits_{x=0}^{r} b(x; n = 15, p = .3)$	.0353	.1268	.9500	$\sum\limits_{x=0}^{r} b(x;15,.2)$	.1671	.3980	.9958
r	1	2	7	r	1	2	7

   a. 0.8232
   b. 0.9147
   c. 0.0853
   d. 0.1768

5. Same background is mentioned as question 4. What is the type II error, β, for the alternative $p = 0.2$.
   a. 0.8287
   b. 0.1713
   c. 0.5978
   d. 0.4022

6. A certain industrial process yields a large number of steel cylinders whose lengths are distributed normally with mean 3.25 inches and standard deviation 0.05 inch. If two such cylinders are chosen at random and placed end to end, what is the probability that their combined length is less than 6.60 inches?
   a. 0.3864
   b. 0.4214
   c. 0.8864
   d. 0.9214

7. Suppose that colored balls are distributed in three boxes as follows:

	Box 1	Box 2	Box 3
Red	2	4	3
White	3	1	4
Blue	5	3	3

A box is selected at random from which a ball is selected at random and it is observed to be red. What is the probability that box 3 was selected?
   a. 1/3
   b. 3/10
   c. 3/28
   d. 9/28

注意：背面尚有試題

8. If a dealer's profit, in units of $1000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & elsewhere. \end{cases}$$

What is the dealer's average profit per automobile if the profit on each automobile is given by $g(X) = X^2$?

a. $147

b. $157

c. $167

d. $177

9. It is same background as above. What probability is the profit will be less than $250 on the next new automobile sold by this dealership.

a. 0.75

b. 0.5

c. 0.4375

d. 0.25

10. Which of the following tests may be used to determine whether a die is balance?

a. t

b. F

c. ANOVA

d. Chi-square

11. A die is tossed 180 times with the following results:

x	1	2	3	4	5	6
f(x)	28	36	36	30	27	23

What is the value of the test statistic?

a. 5.42

b. -2.83

c. 3.36

d. 4.47

12. Suppose that X has distribution N(0, 0.09). A sample of size 25 is obtained from X. What is the probability that $\sum_{i=1}^{25} X_i^2$ exceeds 1.5? $\chi^2_{.95,24} = P(\chi^2 > 13.848) = .95$;

$\chi^2_{.9,24} = 15.659$; $\chi^2_{.8,24} = 18.062$; $\chi^2_{.75,24} = 19.037$; $\chi^2_{.7,24} = 19.943$; $\chi^2_{.5,24} = 23.337$.

a. Between 0.9 and 0.8

b. Greater than 0.9

c. Between 0.75 and 0.8

d. Between 0.5 and 0.7

13. The following sample of size 5 was obtained from the two-dimensional random variable (X, Y). Using these values, compute the sample correlation coefficient.

x	1	2	3	4	5
y	4	5	3	1	2

   a. -0.7

   b. -0.64

   c. -0.8

   d. -0.688

14. In a sample of 80 workers from a factory in city A, it was found that 5% were unable to read, while in a sample of 50 workers in city B, 8% were unable to read. What is the 90% confidence interval for the difference of the two proportions? Will it be concluded that there is a difference in the proportions of nonreader in the two cities under significant level $\alpha = 0.1$?

   a. [-0.105, 0.045]; Yes

   b. [-0.105, 0.045]; No

   c. [-0.026, 0.046]; Yes

   d. [-0.026, 0.046]; No

15. A sample of legal costs (in thousands of dollars) for school districts for two recent consecutive years is shown. What is the 95% confidence interval for the difference in the costs? Will it be concluded that there is a difference in the costs at $\alpha = 0.05$?

$t_{.025,7} = 2.365$; $t_{.025,8} = 2.306$

Year 1	108	36	65	108	87	94	10	40
Year 2	138	28	67	181	97	126	18	67

Part II. There are 40 points in this part. Write down the calculation process as detailed as you can.

16. Let X_1, K, X_n be independent normally distributed random variables, $X_i \sim N(\mu_i, \sigma_i^2)$ and let $Y = \sum_{i=1}^{n} X_i$.

   (a) Show that $Y \sim N(\mu_1 + K + \mu_n, \sigma_1^2 + K + \sigma_n^2)$. (10%)

   (b) Show that if X_i is identical then $\bar{X} \sim N(\mu, \sigma^2 / n)$. (10%)

   Note: The Moment Generating Function of X_i is $M_{X_i}(t) = \exp(\mu_i t + \sigma_i^2 t^2 / 2)$.

17. A small experiment is conducted to fit a multiple regression equation relating the yield y to temperature x1, reaction time x2 and concentration of one of the reactants x3. Two levels of each variable were chosen and measurements corresponding to the coded independent variables were recorded as follows:

y	x1	x2	x3
7.6	-1	-1	-1
8.4	1	-1	-1
9.2	-1	1	-1
10.3	-1	-1	1
9.8	1	1	-1
11.1	1	-1	1
10.2	-1	1	1
12.6	1	1	1

(a) g the coded variables, estimate the multiple linear regression equation

$$\mu_{y|x1,x2,x3} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3.$$ (8%)

(b) Partition SSR (sum square of regression) into three single-degree-of-freedom components attributable to x1, x2 and x3, respectively. Show an analysis-of-variance table, indicating f tests on each variable.   (12%)