

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1710、1720、1740 電腦與通訊研究所甲乙丁組

第一節 工程數學 試題

填准考證號碼

--	--	--	--	--	--	--	--

第一頁 共二頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

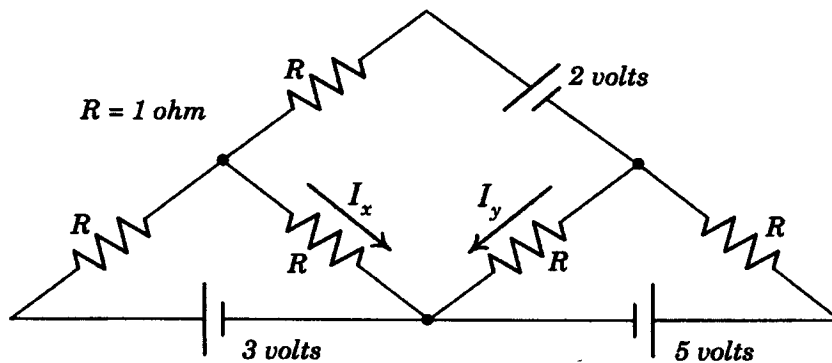
一、Let A and B be the two (4×4) matrices given by

$$A = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 5 & 25 & 125 \\ 1 & 7 & 49 & 343 \end{bmatrix}, \quad B = \begin{bmatrix} -10 & 15 & -20 & 5 \\ 4 & 0 & 0 & 0 \\ -20 & 0 & -0.5 & 0 \\ 8 & 0.2 & 30 & 0 \end{bmatrix}$$

The determinant of an $(n \times n)$ matrix P is denoted as $\det(P)$, then

- (a) Compute $\det(A^T)$. (5%)
- (b) Find $\det(AB^2)$. (5%)

二、Determine the two unknown currents, I_x and I_y , as shown in the following circuit. (15%)



三、 Let W be the subspace of \mathbb{R}^3 , and $W = \{x \mid x = [x_1, x_2, x_3]^T, x_1 + x_2 - 3x_3 = 0\}$. Assume that

$v = [-2, 1, -4]^T$ is in \mathbb{R}^3 .

(a) Find a vector w^* in W such that $(v - w^*)^T \cdot w = 0$ for any vector w in W . (10%)

(b) Calculate the distance between v and w^* . (5%)

四、 A car rental company has two locations, L_a and L_b , at one of which a car is rented freely. A rented car may be returned to either L_a or L_b . The “weekly car distribution history” is listed in the following table.

Weekly Car Distribution		Returned to	
		L_a	L_b
Rented at	L_a	80%	20%
	L_b	30%	70%

Let $v_k = [a(k), b(k)]^T$, where $a(k)$ and $b(k)$ denote the numbers of cars stayed at L_a and L_b , respectively, at the beginning of week k , $k \geq 0$. Initially 500 cars are based at each location, i.e. $v_0 = [500, 500]^T$, and then $v_1 = [550, 450]^T$.

(a) Determine $v_n - v_{n-1}$, $n \geq 1$. (8%)

(b) Find $\lim_{k \rightarrow \infty} \frac{a(k)}{b(k)}$. (7%)

五、 An electronic company uses four assembly lines, L_1, L_2, L_3 , and L_4 , to manufacture 3G mobile phones. The defective rates of mobile phones produced by L_1, L_2, L_3 , and L_4 are 2.5%, 5%, 7.5%, and 10%, respectively. Suppose that 40% of all phones are made by L_1 , while 30%, 20%, and 10% come from L_2, L_3 , and L_4 , respectively. A phone may be marked either qualified or defective.

(a) If a randomly selected phone is defective, what is the probability that it didn't come from L_1 ? (5%)

(b) If a phone selected randomly is qualified, what is the probability that it is made by L_1 or L_4 ? (5%)

注意：背面尚有試題

六、A computer store has purchased 35 PCs of the same type at 550 dollars apiece. It will sell them for 1100 dollars apiece. Any PCs still unsold after one year can be returned to the manufacture at 350 dollars apiece. Let X denote the number of PCs sold. The corresponding *pmf* (probability mass function) of X is defined by

$$p(x; n) = \begin{cases} \frac{2(n-x)}{n(n+1)}, & 0 \leq x \leq n \\ 0, & \text{otherwise} \end{cases}$$

(a) For $p(x; n)$, formulate its *cdf* (cumulative distribution function), $F(x; n)$. (5%)

(b) For $n = 35$, compute $P(0 \leq X \leq 20)$. (5%)

(c) Derive the expected value of X , $E[X]$, $0 \leq x \leq n$. (5%)

(hint: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n \cdot (n+1) = n(n+1)(n+2)/3$.)

Let $h(X) = (\text{revenue} - \text{cost})$ denote the profit associated with selling X PCs.

(d) Calculate $E[h(X)]$, $n = 35$. (5%)

七、Let X and Y be continuous random variables. Then $f(x, y)$ is the joint *pdf* for X and Y , as given by

$$f(x, y) = \begin{cases} kxy, & x, y \geq 0, 1 \leq x+y \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k . (5%)

(b) Derive the marginal *pdf* of X , $f_X(x)$. (5%)

(c) Compute $4 \times P(X+Y \leq 2)$. (5%)