國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1640、1650 電機工程系碩士班丁戊組

第二節 工程數學 試題

填准考證號碼

注意事項：

1. 本試題共十題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. **第一～四題僅需寫答案，不需作答過程。其它題目必須有作答過程。**

一、Let A is an $m\times n$ matrix, and B is an $n\times n$ matrix.

1. (2%) Write the condition of rank(B) if rank(AB) < rank(A).
2. (2%) Write the condition of rank(B) if rank(AB) = rank(A).
3. (2%) Write the condition of rank(B) if rank(AB) > rank(A).

二、(6%) Find the transition matrix A which is the linear

transformation from $\begin{bmatrix}0\\0\\2\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix}$ to $\begin{bmatrix}0\\2\\2\\3\end{bmatrix},\begin{bmatrix}1\\1\\2\\4\end{bmatrix},\begin{bmatrix}4\\2\\3\\1\end{bmatrix}$.

三、(6%) Find all possible matrix X for which $AX=\begin{bmatrix}0\\0\\0\end{bmatrix}$,

where $A=\begin{bmatrix}-2 & 3 & -4\\ 3 & -5 & 6\\ 3 & -7 & 6\end{bmatrix}$.

四、(6%) Let $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \\ 1 \end{bmatrix}$ be one of basis of nullspace of matrix

$\begin{bmatrix} 3 & 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 2 & 1 & 1 & 4 & 1 \\ 3 & 0 & 1 & 2 & 2 \end{bmatrix}$, compute the value of a_1, a_2, a_3, b_1, b_2, b_3.

五、(10%) Let A be a nonsingular $n \times n$ matrix with a nonzero cofactor A_{nn} and set $c = \dfrac{\det(A)}{A_{nn}}$, show that if we subtract c from a_{nn}, then the resulting matrix will be singular.

六、Let $A = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$,

1. (4%) Find the eigenvalues and eigenvectors of A.
2. (12%) Solve the matrix equation $X^2 = A$.

七、Box 1 contains 4 white balls and 4 black balls, and Box 2 contains 2 white balls and 6 black balls. Each time, a ball is picked from a randomly selective box without putting it back.

1. (5 %) Find the probability that the first ball is white.
2. (8 %) Find the probability that the second ball is white. (Note that the first ball and the second ball may come form different boxes)

注意：背面尚有試題

八、X_1, X_2, and X_3 are joint zero-mean Gaussian random variables

with covariance matrix $\Sigma = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, where

$\Sigma_{i,j} = \text{cov}(X_i, X_j)\ \ 1 \le i, j \le 3.$

1. (5 %) Find the joint probability density function of random variable X_1 and X_2.
2. (5 %) Find the conditional probability density function of random variable X_2 given random variable X_1.
3. (5 %) Find the probability density function of random variable $Y = X_1 + X_2 + X_3$.

   (The joint probability density for Gaussian vector $\bar{X} = [X_1, \ldots, X_n]^T$ with mean vector $\bar{\mu}$ and covariance matrix Σ is $f_{\bar{X}}(\bar{x}) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\bar{x}-\bar{\mu})^T \Sigma^{-1}(\bar{x}-\bar{\mu})}$)

九、Let X be a discrete random variable with probability mass function

$$f_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, \ldots, n \\ 0 & \text{else} \end{cases} \quad \text{for some } 0 < p < 1$$

1. (4 %) Show that the moment-generating function of the random variable X is given by

$$M_X(t) = E\left[e^{tx}\right] = \left(pe^t + (1-p)\right)^n$$

2. (6 %) Using the moment-generating function in part (a), find $E[X]$ and $E[X^2]$.

十、Let X and Y be independent random variables with probability density function

$$f_X(x)=\begin{cases} e^{-x} & x\ge 0 \\ 0 & x<0 \end{cases},\quad f_Y(y)=\begin{cases} 2e^{-2y} & y\ge 0 \\ 0 & y<0 \end{cases}$$

1. (6 %) Find the probability density function of random variable $Z_1=2X+Y$
2. (6 %) Find the probability density function of random variable $Z_2=X/Y$.