ee11-1

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1610、1620、1630 電機工程系碩士班甲乙丙組

第二節 工程數學 試題

填准考證號碼

第一頁 共一頁

注意事項：

1. 本試題共十題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. **第一～五題僅需寫答案，不需作答過程。其它題目必須有作答過程。**

一、Let A is an $m\times n$ matrix, and B is an $n\times n$ matrix.

1. (2%) Write the condition of rank(B) if rank(AB) < rank(A).
2. (2%) Write the condition of rank(B) if rank(AB) = rank(A).
3. (2%) Write the condition of rank(B) if rank(AB) > rank(A).

二、(6%) Find the transition matrix A which is the linear

transformation from $\begin{bmatrix}0\\0\\2\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix}$ to $\begin{bmatrix}0\\2\\2\\3\end{bmatrix},\begin{bmatrix}1\\1\\2\\4\end{bmatrix},\begin{bmatrix}4\\2\\3\\1\end{bmatrix}$.

三、(6%) Find all possible matrix X for which $AX=\begin{bmatrix}0\\0\\0\end{bmatrix}$,

where $A=\begin{bmatrix}-2 & 3 & -4\\ 3 & -5 & 6\\ 3 & -7 & 6\end{bmatrix}$.

四、(6%) Let $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \\ 1 \end{bmatrix}$ be one of basis of nullspace of matrix

$\begin{bmatrix} 3 & 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 2 & 1 & 1 & 4 & 1 \\ 3 & 0 & 1 & 2 & 2 \end{bmatrix}$, compute the value of a_1, a_2, a_3, b_1, b_2, b_3.

五、(6%) Find the interval of convergence of the following series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 3^n}(x-2)^n$$

六、(10%) Let A be a nonsingular $n \times n$ matrix with a nonzero cofactor A_{nn} and set $c = \dfrac{\det(A)}{A_{nn}}$, show that if we subtract c from a_{nn}, then the resulting matrix will be singular.

七、Let $A = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$,

1. (4%) Find the eigenvalues and eigenvectors of A.
2. (12%) Solve the matrix equation $X^2 = A$.

八、(15%) Solve $y'' + 9y = -4x\cos(3x)$

九、(15%) Use the Laplace transform to solve

$$t\,y'' + (4t+2)\,y' - 4\,y = 0; \qquad y(0) = 2$$

十、For $x^2 y'' + x y' + (x^2-1) y = 0$,find

1. (4%) indicial equation
2. (10%) two linearly independent solutions.