國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1820 資訊工程系碩士班乙組

第一節 工程數學 試題

填准考證號碼

注意事項：

1. 本試題共 9 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Essay questions (15 %)
   a. Let an experiment have sample space S and the probability associated with an event E be $P(E)$. Give the three axioms that $P(E)$ must satisfy.
   b. Can two events both independent and mutually exclusive? Why?
   c. Some random variables do not have variances. Give one example by specifying its name and density function.
2. A medical doctor faces the following dilemma: "If I am at least 80 % certain that my patient has disease D, then I always recommend surgery. Otherwise, I recommend additional tests. Initially, I was 60 % certain that my patient had disease D, so I ordered test T. The result of test T is positive (i.e., having disease D). I know that test T is also sensitive to the blood type of the person under test. Test T always gives the correct result if blood type of the person under test is not B. However, the test has 30 % of chance to falsely indicate a healthy person with blood type B to have disease D. After the test, I realized that my patient has the blood type B. What should I do? Immediate surgery or more tests?" Answer the question solely based on your calculated probability. (8 %)
3. In the wafer process, the number of defects on a chip affects its yield rate. A chip will fail if it has one or more defects on it. Assume that a 100-mil^2 wafer contains 50 defects and the number of defects in a small area follows the Poisson distribution. If the wafer is to be cut into many chips of equal size, find the maximum size of each chip so that the probability of chip failure is less than 20 percent. You may assume that the wafer area can be cut into an integer number of chips without fragments. (7 %)
4. The exponential random variable X has the following density function

$$f(x) = \lambda e^{-\lambda x}, \lambda > 0, x > 0.$$

Find $E[X^n], n > 0$. (10 %)

5. It is known that the random variables X and Y have the following joint density function

$$f(x,y)=\begin{cases}1, & 0<x\le 2,\ 0<y\le 0.5\\ 0, & \text{otherwise}\end{cases}$$

Find the density function of random variable $Z=X+Y^2$. (10 %)

6. Let the matrix A be as follows. (20 %)

$$A=\begin{bmatrix}10 & -9 & 0 & 0\\ 4 & -2 & 0 & 0\\ 0 & 0 & 1 & 2\\ 0 & 0 & 4 & 3\end{bmatrix}$$

a. Find eigenvalues of A.

b. Find the determinant of A^2.

c. Find A^{-1}.

d. Determine if A^TA is invertible. Give reasons.

7. It is known that

$$A=\begin{bmatrix}2 & 2 & -1 & 0 & 1\\ -1 & -1 & 2 & -3 & 1\\ 1 & 1 & -2 & 0 & -1\\ 0 & 0 & 1 & 1 & 1\end{bmatrix}.$$

Find an orthonormal basis for the vector space $\mathcal{V}=\{\mathbf{x}\mid A\mathbf{x}=\mathbf{0},\mathbf{x}\in \mathbf{R}^5\}$ with the Euclidean inner product. (10 %)

8. Show that if A is a real symmetric matrix, then the eigenvectors associate with distinct eigenvalues of A are orthogonal. (10 %)

9. Let A be a complex matrix and $A^*=-A$ where "*" denotes the conjugate transpose. Show that $(I-A)^{-1}$ exists. (10 %)