國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1810 資訊工程系碩士班甲組

第二節 離散數學與演算法 試題

填 准 考 證 號 碼

注意事項：

1. 本試題共八題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、(10 pts) True or False. If the statement is true, please give the reasons; otherwise, give a counter example. Assume that the functions f, g, and h take on only positive values. (Please note that, only giving "true" or "false" without explanation will get no point)

1. If $f(n)=\Theta(h(n))$ and $g(n)=\Theta(h(n))$, then $f(n)+g(n)=\Theta(h(n))$.
2. If $f(n)=\Theta(g(n))$, then $2^{f(n)}=\Theta(2^{g(n)})$.
3. If $f(n)=O(g(n))$, then $g(n)=O(f(n))$.
4. If $f(n)=O(g(n))$, then $g(n)=\Omega(f(n))$.
5. $f(n)+g(n)=\Theta(h(n))$, where $h(n)=\min\{f(n), g(n)\}$.

二、(15 pts) Please select the correct answer for each of the following questions.

1. (5 pts) Assuming that p is true, q is false, and r is true, which one is false?
   (A) $(p \wedge q) \rightarrow r$
   (B) $(p \vee q) \rightarrow \overline{r}$
   (C) $p \wedge (q \rightarrow r)$
   (D) $p \rightarrow (q \rightarrow r)$

2. (5 pts) Let x_n be a sequence satisfying $x_{n+1}=3x_n-2x_{n-1}$ with $x_0=2$, $x_1=3$, what is x_n?
   (A) $n+2$
   (B) n^2+2
   (C) 2^n+1
   (D) 2^n-1

3. (5 pts) How many times is the `print` statement executed?

```
for i1:= 1 to n do
  for i2 := 1 to i1 do
    for i3 := 1 to i2 do
          .
            .
              .
                for ik := 1 to ik-1 do
                   print i1, i2, …, ik
```

(A) $C(n-1,k)$
(B) $C(k+n,n)$
(C) $C(k+n-1,k)$
(D) $C(n,k)$

三、(10 pts) Prove the following equation by induction

$$\sum_{i=1}^{k} 2^{i-1}(k-i)=2^k-k-1,$$

for $k \geqq 1$.

四、(10 pts) Consider that a algorithm divides a problem of size n into two subproblems, each of size $\sqrt{n}$. These two subproblems are then solved recursively by calling the algorithm itself. Suppose the algorithm takes O(log n) time to divide and merge the subproblems. Please denote the running time T(n) using recurrence and show the asymptotic upper bound for the running time T(n) as tight as possible. Assume that T(n) is constant when $n \leqq 4$.

五、(5 pts) For the grammar listed below, please construct a finite state machine that accepts the sentences in the language specified by the grammar.

A → 0A
A → 1B
B → 0C
B → 0D
C → 0
C → 1B
C → 1D
D → 1
D → 1A

注意：背面尚有試題

六、(15 pts) Let (**B**,∧,∨,¬, 1, 0) be a Boolean algebra. Define the operation ⊕ in the Boolean algebra as $p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q)$.

1. Show that $p \oplus q$ is equivalent to $(p \vee q) \wedge \neg(p \wedge q)$ (5pts)
2. Does (**B**, ⊕) form an abelian group? Show your reasons. (10pts)

七、(10 pts) Let A be an array of n arbitrary and distinct numbers. A has the following property: If we imagine B as being sorted version of A, then any element that is at position i in array A would, in B, be at a position j such that $|i-j| \leq k$. In other words, each element in A is not farther than k positions away from where it belongs in the sorted version of A. Suppose you are given such an array A, and you are told that A has this property for a particular value k (that value of k is also given to you). Design an O($n$log k) time algorithm for sorting A.

八、(25 pts) Consider the edge-weighted connected graph $G = (V, E)$ in Figure 1 where V is the vertex set and E is the edge set of G respectively.

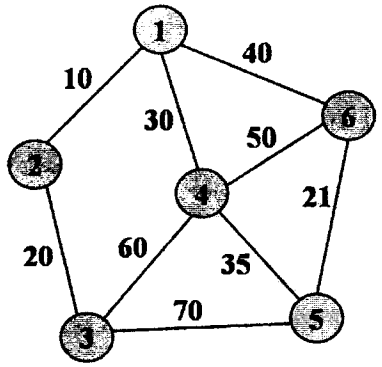

Figure 1: The edge-weighted connected graph G.

1. (5 pts) Please find a minimum-cost spanning tree of G by Kruskal's algorithm. Please show your work step by step.
2. (10 pts) Write down the pseudo-code of the Kruskal's algorithm and show that the time complexity of the Kruskal's algorithm is O($|E|$ log $|E|$).
3. (10 pts) Please prove that Kruskal's algorithm generates a minimum-cost spanning tree for every connected undirected graph.