

## 國立臺北科技大學九十五學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

## 第三節 工程數學 試題

填准考證號碼

第一頁 共一頁

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**注意事項：**

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) Determine the constants A, B, C, D of the Laplace transform of function  $f(t)$

$$L[f(t)] = \frac{As^3 + Bs^2 + Cs + D}{(s^3 + 6s^2 + 13s)^2}, \text{ where } f(t) = t \int_0^t e^{-3\tau} \cos 2\tau d\tau$$

2. (25%) A diffusion problem can be described by the following partial differential equation:

$$\alpha^2 \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t} \quad 0 \leq r < c, \quad 0 < t < \infty$$

$$u(c, t) = 100, \quad u(0, t) \text{ is bounded}$$

$$u(r, 0) = 0$$

Find the solution of  $u(0, t)$

3. (10%) The equation

$$\frac{d}{dx} \left( x^a \frac{dy}{dx} \right) + bx^c y = 0$$

where  $a, b, c$  are real numbers and  $b > 0$ , can be transformed to the Bessel equation

by  $t = \alpha \sqrt{bx}^{1/\alpha}$ ,  $u = x^{-\nu/\alpha} y$ . And the solution to the Bessel equation is

given as the form

$$y(x) = x^{\nu/\alpha} [AJ_\nu(\alpha \sqrt{bx}^{1/\alpha}) + BY_\nu(\alpha \sqrt{bx}^{1/\alpha})]$$

where  $\alpha = 2/(c - a + 2)$ ,  $\nu = (1 - a)/(c - a + 2)$  and A, B are constants.

Solve the equation  $y'' + 3\sqrt{x}y = 0 \quad (0 < x < \infty)$

4. (20%) Use the Laplace transform to solve the following differential equation

$$ty'' + (t-1)y' + y = 0, y(0)=0$$

5. (15%) Use Green's theory to evaluate  $\oint_C \vec{F} \cdot d\vec{r}$ ,  $C$  is a oriented counterclockwise circle of

radius 2 about  $(-8,0)$ , where  $\vec{F} = (e^{\sin x} - y)\vec{i} + (\sinh(y^3) - 4x)\vec{j}$

6. (15%) Find the flux of  $\vec{V} = x\vec{i} + y\vec{j} - z\vec{k}$  across the part of the plane  $x + 2y + z = 8$  lying in the first octant.