ATY3-1

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1411、1412、1421、1422 能源與冷凍空調工程系碩士班

甲乙組

第一節 工程數學 試題

填准考證號碼 [ ][ ][ ][ ][ ][ ][ ][ ]

第一頁 共一頁

注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、 1. Let $M(x,y)$ and $N(x,y)$ be defined on a region R of the plane. Then $\mu(x,y)$ is an integrating factor for $M + N\dfrac{dy}{dx} = 0$ if $\mu(x,y) \neq 0$ for all (x,y) in R, and $\mu M + \mu N\dfrac{dy}{dx} = 0$ is exact on R. Therefore, we can get

$$\frac{\partial}{\partial x}(\mu N) = \frac{\partial}{\partial y}(\mu M) \qquad (1)$$

Please use Equation (1) to find the integrating factor of a first-order linear differential equation: $\dfrac{dy(x)}{dx} + p(x)y = q(x)$. (10%)

2. Find the solution of $\dfrac{dy}{dx} - (\tan x)y = e^{\sin x}$, $y(0)=1$. (10%)

二、 Please find the solutions of the following equations:

1. $\dfrac{dx}{dy} = \dfrac{(x-y)}{(x+y-2)}$ (10%)
2. $\dfrac{dy}{dx} + \dfrac{x-2y+3}{2x-4y+5} = 0$ (10%)

三、 Please find the solutions of the following equations by the method of Laplace transform:

1. $y'' + 2y' + 2y = r(t)$; $r(t) = 10\sin 2t$ if $0 < t < \pi$ and 0 if $t > \pi$; $y(0) = 1$, $y'(0) = -5$. (10%)
2. $y'' + 5y' + 4y = 0$, $y(0) = 1$, $y(1) = 0$ (10%)

四、A differential equation is shown in Equation (2)

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \qquad (2)$$

where m is a real constant.

1. If the boundary conditions are given by

$$\theta(0) = \theta_0, \ \theta(L) = \theta_L$$

, please prove that the solution of Equation (2) is

$$\frac{\theta}{\theta_0} = \frac{(\theta_L/\theta_0)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

(10%)

2. If the boundary conditions are given by

$$\theta(0) = \theta_0, \ h\theta(L) = -k\left.\frac{d\theta}{dx}\right|_{x=L}$$

, please prove that the solution of Equation (2) is

$$\frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}.$$

(10%)

Note: In 1 and 2, h, k, L, θ_0, and θ_L are real constants.

Also, sinhx and coshx are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

五、Suppose that the functions p, q, r, and $\frac{dr}{dx}$ in the Sturm-Liouville equation, as shown in equation (3), are real-valued and continuous and $p(x)>0$ on the interval $a \le x \le b$.

$$\frac{d}{dx}\left[r(x)\frac{dy}{dx}\right] + [q(x) + \lambda p(x)]y = 0 \qquad (3)$$

Let $y_m(x)$ and $y_n(x)$ be eigenfunctions of the Sturm-Liouville equation that correspond to different eigenvalues λ_m and λ_n, respectively. The solutions of equation (3) must satisfy regular boundary conditions

$$A_1 y(a) + A_2 y'(a) = 0, \ B_1 y(b) + B_2 y'(b) = 0.$$

A_1 and A_2 are given constants, not both zero, and similarly for B_1 and B_2.

Please prove that $y_m(x)$ and $y_n(x)$ are orthogonal on the interval $[a,b]$ with respect to the weight function $p(x)$, that is,

$$\int_a^b p(x)y_m(x)y_n(x)dx = 0 \qquad m \ne n.$$

(20%)