

國立臺北科技大學

九十四學年度機電整合研究所入學考試

工程數學試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共七題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

*** 參考積分公式 ***

$$\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{u^2+a^2}}{u} \right| + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$$

$$\int e^{au} \sin bu = \frac{e^{au}(a \sin bu - b \cos bu)}{a^2+b^2} + C$$

$$\int e^{au} \cos bu = \frac{e^{au}(a \cos bu + b \sin bu)}{a^2+b^2} + C$$

1. Refer to Fig. 1, if the relation between before and after the axis rotation can be represented by

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \mathbf{T} \cdot \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Find the orthonormal transformation matrix \mathbf{T} when a CCW rotation θ is applied to x -axis. (5%)

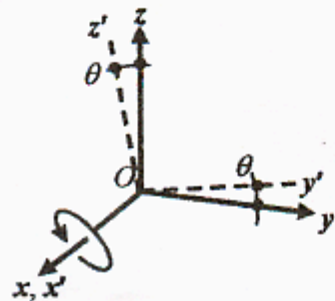


Fig. 1

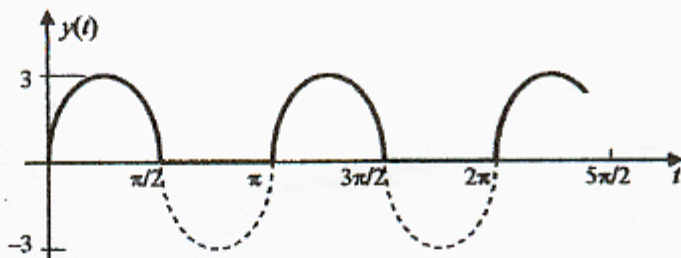


Fig. 2

2. A periodic function $y(t)$ can be represented by the rectified sine function as shown in Fig. 2.

Let $u(t)$ denote the unit-step function, then

(a) Write $y(t)$ in terms of unit step functions; (5%)

(b) Find the Laplace transform of $y(t)$. (10%)

3. If $x(t)$ and $y(t)$ satisfy the differential equations

$$\begin{cases} x' - y' + x = 5 \\ x'' - y' + 3x - y = e^{3t} \end{cases} \quad \text{where ' and '' denote } \frac{d}{dt} \text{ and } \frac{d^2}{dt^2}, \text{ respectively. What}$$

are $x(t)$ and $y(t)$? (20%)

4. Find a solution for $f(x, y)$ that satisfies the partial differential equation

$$x \frac{\partial f}{\partial x} + 2y \frac{\partial f}{\partial y} = 0. \quad (15\%)$$

5. A vector field F is defined by $F = 2x\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k}$. Refer to Fig. 3, if one moves a particle along path C starting from O to D . Let $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be a position vector from O .

(a) Please compute $\int_C F \cdot dr = ?$ (10%)

(b) If path C is reverse, i.e., $O \rightarrow D \rightarrow B \rightarrow A$, and back to O , then $\int_C F \cdot dr = ?$ (5%)

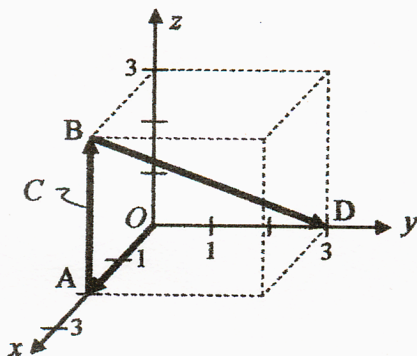


Fig. 3

6. If a matrix A is defined as

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix},$$

Find all the eigenvalues of A . (15%)

7. Solve the differential equation $xy'' - 2\sqrt{1+(y')^2} = 0$ for $y(x)$. (15%)