國立臺北科技大學

九十四學年度電腦與通訊研究所入學考試

工程數學（甲乙丁組）試題

填准考證號碼 第一頁 共一頁

注意事項：

1. 本試題共七題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

一、(36%) Answer each of the following questions as true (T) or false (F). If your answer is false (F), please explain why it is wrong or write down the correct answer.

(a) The sets A_1, A_2, and A_3 are independent if and only if

$$P[A_1 \cap A_2 \cap A_3] = P[A_1] \cdot P[A_2] \cdot P[A_3]$$

(b) If random variable $Y = a \cdot X$ and the standard deviations of X and Y are σ_X and σ_Y, Then $\sigma_Y = a \cdot \sigma_X$

(c) Let $f_{Y|X}(y \mid x)$ be the conditional PDF of Y given X=x, then the joint PDF of X and Y can be expressed as $f_{X,Y}(x, y) = f_{Y|X}(y \mid x) \cdot f_X(x) = f_{X|Y}(x \mid y) \cdot f_Y(y)$

(d) Consider a binary code with 4 bits (0 or 1) in each code word. The probability of the event that each code word has exactly two zeros is equal to 0.5.

(e) X is the temperature of a resistor measured in degree Celsius (^oC). Y is the temperature of the same resistor measured in degree Kelvin (^oK). The correlation coefficient of the two random variables X and Y is equal to 1.

(f) If X_1 and X_2 are independent Bernoulli random variables and $Y=X_1+X_2$, Then Y is a binomial random variable.

(g) If a subset of a vector space does not include the zero vector, that subset cannot be a subset.

(h) If A is an $n \times n$ matrix, then $|3 \cdot A| = 3 \cdot |A|$. ($|A|$: determinant of matrix A)

(i) The sum of two eigenvectors of a matrix A is also an eigenvector of A.
(j) If V is orthogonal to W and W is orthogonal to Z, Then V is orthogonal to Z.
(k) If the only eigenvectors of A are multiples of x=(1, 0, 0), then A is not invertible.
(l) Suppose S and T are subspace of R^{13}, with dimension dim S=7, dim T=8. The smallest possible dimension of $(S \cap T)$ is 7.

二、(10%) W is a Gaussian random variable with mean μ=0, and variance Var[W]=4. Given the event C={W>0}

(1) Find the conditional expected value, E[W|C]. (5%)

(2) Find the conditional variance, Var[W|C]. (5%)

三、(10%) Telephone calls can be classified as "voice" (V) if someone is speaking or data (D) if there is a modem or fax transmission. Given P[V]=3/4 and P[D]=1/4, and the random variable Kn is the number of voice calls in a collection of n phone calls. What are the expected value and standard deviation of K_{48}.

四、(12%) Let $X_1, X_2, \ldots, X_n$ be random variables with the same mean μ and with covariance function

$$COV(X_i, X_j) = \begin{cases} \sigma^2 & \text{if } i = j \\ \rho\sigma^2 & \text{if } |i - j| = 1 \\ 0 & \text{otherwise} \end{cases}$$

where $|\rho| < 1$. Find the mean and variance of $S_n = X_1 + X_2 + \ldots + X_n$

五、(12%) If $A = \begin{bmatrix} 7 & 12 \\ -4 & -7 \end{bmatrix} = PDP^{-1}$, find $P, D,$ and A^{-101}.

六、(10%) Let the vector space P_2 have the inner product

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$$

Apply the Gram-Schmidt process to transform the standard basis S={1, x, x^2} into an orthonormal basis.

七、(10%) Find the 4 by 4 matrix A that represents a cyclic permutation: each vector (x_1, x_2, x_3, x_4) is transformed to (x_2, x_3, x_4, x_1). What is the effect of A^2? Show that $A^3 = A^{-1}$.