

國立臺北科技大學

九十四學年度電機工程系碩士班入學考試

工程數學（丁戌組）試題

填准考證號碼

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注意事項：

1. 本試題共七題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Box 1 contains 100 bulbs of which 10% are defective. Box 2 contains 100 bulbs of which 5% are defective. Two bulbs are picked from a randomly selective box.

(a) (5 %) Find the probability that both bulbs are defective.

(b) (5 %) Assuming that both bulbs are defective, find the probability that they came from box 1.

2. The joint probability density function of the random variables X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{3x-y}{9} & 0 < x < 3, 1 < y < 2 \\ 0 & \text{else} \end{cases}$$

(a) (5 %) Find the marginal probability density function of random variable Y.

(b) (5 %) Find the conditional probability density function of random variable X given random variable Y.

3. Let X be a random variable with probability density function

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

for some $\lambda > 0$.

(a) (6 %) Show that the moment-generating function of the random variable X is given by

$$M_X(t) = E[e^{tx}] = \frac{\lambda}{\lambda - t} \quad \text{for } t < \lambda$$

(b) (8 %) Let random variable $Y = X_1 + 2X_2 + 3X_3$, where X_1, X_2, X_3 are independent random variables with the same probability density function $f_X(x)$. Find $E[Y]$ and $E[Y^2]$.

4. The random variables X and Y have the joint density

$$f_{XY}(x, y) = \begin{cases} 1 & 0 < x < 1, 0 < y < 1 \\ 0 & \text{else} \end{cases}$$

(a) (5 %) Find the probability density function of random variable $Z_1 = \max(X, Y)$.

(b) (6 %) Find the probability density function of random variable $Z_2 = \min(X, Y)$.

(c) (5 %) Find $P(Z_1 < \frac{2}{3}, Z_2 < \frac{1}{3})$.

5. (15%) Let $B = \begin{bmatrix} 6 & -2 & -4 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$, find $B^m = ?$ (m is a positive integer)

6. (20%) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$, find an orthonormal basis for the

column space of A .

注意：背面尚有試題

7. (15%) Let $E = [u_1, u_2, u_3]$ and $F = [b_1, b_2, b_3, b_4]$, where

$$u_1 = (1, 2, 1)^T, \quad u_2 = (-1, 1, 1)^T, \quad u_3 = (1, 0, -2)^T$$

$$\text{and } b_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad b_4 = \begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}.$$

Find the matrix representing L with respect to the ordered

bases E and F , where $L(\mathbf{x}) = \begin{bmatrix} x_1 + x_2 & x_1 - x_3 \\ x_2 - x_3 & x_2 + x_3 \end{bmatrix}$.