國立臺北科技大學

九十四學年度資訊工程系碩士班入學考試

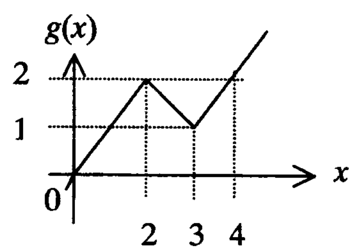
工程數學試題

填准考證號碼　　　　　　　　　　　　　　

注意事項：

1. 本試題共 8 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Answer the following questions as true or false. You MUST give reasons to justify your answer.
   a. If $f(x,y)$ is a joint pdf, then $f(x,y) \geqq 0$ for all x and y. [4 %]
   b. The Law of Large Number says that the sum of a lot of independent identical random variables has a Gaussian-like distribution. [4 %]
   c. The third moment of a zero-mean, unit-variance Gaussian random variable is zero. [4 %]
2. Essay questions.
   a. What is a random variable? [4 %]
   b. What is the definition of independent events? [4 %]
   c. Briefly explain the Bayes' rule. [4 %]
3. Let X be an exponential random variable with pdf $f(x)=e^{-x}, x \geq 0$. Let $Y = g(X)$ be another random variable with $g(x)$ shown below. Find the pdf of the random variable Y. [13 %]



4. Let X, Y, and Z be independent zero-mean, unit-variance Gaussian random variables. Find the pdf of the random variable $W = \sqrt{X^2 + Y^2 + Z^2}$. [13 %]

5. Determine if the inverse of each of the following $n \times n$ matrices exist. Explain your reasons.

   a. Matrix A is skew-symmetric, i.e., $A^{\mathrm{T}} = -A$, with odd n. [4 %]

   b. A is nilpotent, i.e., $A^k = 0$ for $k \geqq 1$. [4 %]

   c. $A = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$. [8 %]

6. Essay questions.

   a. Give the definition of the dimension of a vector space. [4 %]

   b. If $T: V \rightarrow W$ is a linear transformation, give the definition of the range of T. [4 %]

7. Let V and W be subspaces of a space F. Is $V \cap W$ a subspace of F? Why? [13 %]

8. Evaluate det(A) given that A is an $n \times n$ matrix shown below (blanks in the matrix represent zeros): [13 %]

$$A = \left[\begin{array}{ccc|c} 1 & & & 1 \\ & \ddots & & \vdots \\ & & 1 & 1 \\ \hline 1 & \cdots & (n-1) & 1 \end{array}\right]$$