國立臺北科技大學

九十四學年度化學工程研究所入學考試

工程數學試題

填准考證號碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1、A heat transfer system govern by the following equations

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}, \quad -L \le y \le L$$

and

$t = 0$, $T = T_0$

$y = 0$, $\dfrac{\partial T}{\partial y} = 0$

$y = \pm L$、$-k\dfrac{\partial T}{\partial y} = h(T - T_\infty)$

a) Change the variables to $\Phi = \dfrac{T - T_\infty}{T_0 - T_\infty}$, $\eta = y/L$, $\theta = \dfrac{\alpha t}{L^2}$. (10%)

b) Solve the dimensionless partial differential equation with appropriate initial and boundary conditions. (if the eigen value is the roots of a nonlinear equation, no need to solve the eigen values explicitly). (10%)

2、Solve the following partial differential equations by Laplace transforms

$$\frac{\partial v}{\partial x} + 2x\frac{\partial v}{\partial t} = 2x,\ v(x,0)=1,\ v(0,t)=1.\ (20\%)$$

3、Evaluate a line integral $\int_C \vec{F}\cdot\vec{r}'(s)ds$ by Stokes's theorem $\iint_S (\nabla\times\vec{F})\cdot\vec{n}dA$,

where C is the circle $x^2+y^2=a^2$, z=b, which is the boundary of S and

$\vec{F} = y\vec{i} + xz^3\vec{j} - zy^3\vec{k}$. (20%)

4、$$\frac{\partial C_a}{\partial t} = \frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial C_a}{\partial r}\right]$$

B. C. 1, $r=0$, $C_a=0$, for any t

B. C. 2, $r=\infty$, $C_a=C_{a\infty}$, for any t

a) By use of $\eta = \frac{r}{t^{2/3}}$, change the partial differential equation to an ordinary differential equation, expressed as

$$A\eta^C\frac{dC_a}{d\eta} = \frac{d}{d\eta}\left(\eta^B\frac{dC_a}{d\eta}\right)$$

Find the constants A, B, C. (10%)

b) Let $z=\eta^B$, Solve $\frac{C_a}{C_{a\infty}}$ from the derived ordinary differential equation with appropriate boundary conditions as shown above.(10%)

5、Show the Fourier Cosine Integral Representation of the function

$f(x)=e^{-x}+e^{-2x}$ x>0 has the form of $\frac{6}{\pi}\int_0^\infty \frac{A+\omega^2}{B+C\omega^2+D\omega^4}\cos\omega x d\omega$.

Determine A, B, C, and D. (20%)