國立臺北科技大學

九十四學年度光電工程系碩士班入學考試

電磁學試題

填准考證號碼　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

Useful constants and formula:

$$\varepsilon_0 = 10^{-9}/(36\pi)\ (F/m),\ \ \mu_0 = 4\pi\times 10^{-7}(H/m)$$

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$$

1. (25%)

A coaxial transmission line using the dielectric medium of permittivity ε has a solid inner conductor of radius $\boldsymbol{a}$ and a very thin outer conductor of inner radius $\boldsymbol{b}$.

(1) Derive the inductance per unit length of this transmission line from electrostatics. (6%)

(2) Derive the capacitance per unit length of this transmission line from magnetostatics. (8%)

(3) Assume this transmission line is lossless. Determine its characteristic impedance at high frequency. (6%)

(4) The dielectric medium is polyethylene (ε_r=2.25). Find the inner radius of the outer conductor for a 50Ω coaxial transmission line, where the radius of the inner conductor is 0.6mm. (5%)

2. (20%)

An electromagnetic wave has the electric field intensity:

$$\vec{\mathrm{E}} = \hat{\mathrm{a}}_{\mathrm{x}}\cdot 100\cdot\cos\left(4.9\times 10^{9}\cdot\pi\cdot t - k\cdot z + \theta\right)\quad \mathrm{(V/m)}$$

(1) When this wave propagates in free space, determine its magnetic field intensity $\vec{H}$ (5%)

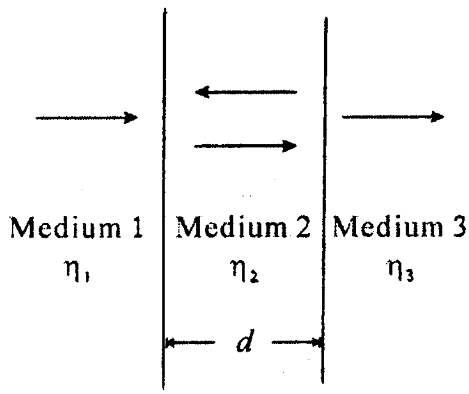
and the average Poynting vector. (5%)

(2) A lossy dielectric medium has a relative permittivity of 36 and a loss tangent of 0.4, find its effective conductivity at the frequency of this electromagnetic wave. (5%)

(3) When this electromagnetic wave exists in the lossy medium of (2), find the average power dissipated in this medium per cubic meter. (5%)

3. (10%)

A plane wave of wavelength λ is normally incident on a dielectric film of index n and thickness d, as shown in the following figure. The medium i (i=1, 2, 3) has an intrinsic impedance η_i. Consider two cases: (1) $\eta_1 = \eta_3$; (2) $\eta_1 \neq \eta_3$. Determine the intrinsic impedance η_2 and thickness d such that no reflection occurs. (10%)



4. (15%)

Prove the following equation:

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \cdot \int_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) \cdot dv - \int_V \sigma E^2 \cdot dv$$

5. (15%)

For a lossy transmission line at very low frequency such that $\omega L \ll R$, and $\omega C \ll G$, derive approximate expressions for α, β (propagation constant $\gamma = \alpha + j \cdot \beta$) and R_0, X_0 (characteristic impedance $Z_0 = R_0 + j \cdot X_0$) in terms of ω, R, L, G, C.

6. (15%)

In a waveguide, the electric and magnetic fields can be divided into the transverse and longitudinal components, i.e. $\vec{E} = \vec{E}_T + \hat{a}_z E_z$, $\vec{H} = \vec{H}_T + \hat{a}_z H_z$, $\nabla = \nabla_T + \hat{a}_z \cdot \partial / \partial z$, where the subscript T denotes "transverse". The transverse field components can be expressed in terms of the longitudinal components. Prove the following relations for time harmonic excitation:

(1) $\vec{E}_T = -\frac{1}{h^2}\left(\gamma \cdot \nabla_T E_z - \hat{a}_z \cdot j\omega\mu \times \nabla_T H_z\right)$

(2) $\vec{H}_T = -\frac{1}{h^2}\left(\gamma \cdot \nabla_T H_z + \hat{a}_z \cdot j\omega\varepsilon \times \nabla_T E_z\right)$

where $h^2 = \gamma^2 + k^2$, γ is the propagation constant ($\gamma = \alpha + j \cdot \beta$), and $k = \omega\sqrt{\mu \cdot \varepsilon}$