

國立臺北科技大學

九十三年學年度車輛工程系碩士班入學考試

工程數學試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共六題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Given an adjacency matrix $adj A = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, Calculate the determinant $|A|$ and the matrix A , respectively. (15 %)

2. Solve the following differential equation. (15%)

$$x^2 y'' - 5xy' + 8y = 2x \ln(x) + x^3$$

3. Solve the following first order O.D.E. system. (15%)

$$\frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \text{ with } \begin{bmatrix} x(0) \\ y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

4. Solve the following first order O.D.E. (15%)

$$x dy - \{y + xy^3(1 + \ln x)\} dx = 0$$

5. Solve the following second O.D.E. (20%)

$$y'' + 3y' + 2y = f(x) \quad , \quad y(0) = y'(0) = 0.$$

Where $f(x)$ can be described as $f(x) = \begin{cases} 0, & 0 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$ with $f(x+2) = f(x)$.

6. Solve the following the P.D.E

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = -2, \quad 0 < r < a, \quad 0 < \theta < 2\pi$$

Boundary conditions: $u(r, \theta) = u(r, \theta + 2\pi)$, $\frac{\partial u(r, \theta)}{\partial r} = \frac{\partial u(r, \theta + 2\pi)}{\partial r}$.

- (a) Prove that the homogeneous solution of $\nabla^2 u$ can be described as

$$u_H(r, \theta) = A_0 + B_0 \ln r + \sum_{n=1}^{\infty} \left[(A_n r^n + B_n r^{-n}) \cos n\theta + (C_n r^n + D_n r^{-n}) \sin n\theta \right] \quad (10\%)$$

- (b) If the P.D.E has an additional boundary condition $u(a, \theta) = 0$, determine a general solution of the P.D.E. (10%)