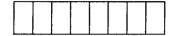
國立臺北科技大學

九十三學年度車輛工程系碩士班入學考試

工程數學試題

填准考證號碼

第一頁 共一頁



- 注意事項:
 1. 本試題共六題,配分共100分。
 2. 請按順序標明題號作答,不必抄題。
- 全部答案均須答在答案卷之答案欄內,否則不予計分。
- 1. Given an adjacency matrix $adj \mathbf{A} = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, Calculate the determinant $|\mathbf{A}|$ and the matrix A, respectively. (15 %)
- 2. Solve the following differential equation. (15%)

$$x^2y'' - 5xy' + 8y = 2x\ln(x) + x^3$$

3. Solve the following first order O.D.E. system. (15%)

$$\frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \text{ with } \begin{bmatrix} x(0) \\ y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

4. Solve the following first order O.D.E. (15%)

$$xdy - \{y + xy^3(1 + \ln x)\}dx = 0$$

5. Solve the following second O.D.E. (20%)

$$y'' + 3y' + 2y = f(x)$$
, $y(0) = y'(0) = 0$.

Where
$$f(x)$$
 can be described as $f(x) = \begin{cases} 0, & 0 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$ with $f(x+2) = f(x)$.

6. Solve the following the P.D.E

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = -2, \quad 0 < r < a, \quad 0 < \theta < 2\pi$$

Boundary conditions:
$$u(r,\theta) = u(r,\theta+2\pi)$$
, $\frac{\partial u(r,\theta)}{\partial r} = \frac{\partial u(r,\theta+2\pi)}{\partial r}$.

(a) Prove that the homogeneous solution of $\nabla^2 u$ can be described as

$$u_{H}(r,\theta) = A_{0} + B_{0} \ln r + \sum_{n=1}^{\infty} \left[\left(A_{n} r^{n} + B_{n} r^{-n} \right) \cos n\theta + \left(C_{n} r^{n} + D_{n} r^{-n} \right) \sin n\theta \right]$$
 (10%)

(b) If the P.D.E has an additional boundary condition $u(a,\theta) = 0$, determine a general solution of the P.D.E. (10%)