

# 國立臺北科技大學

## 九十三年學年度機電整合研究所入學考試

### 工程數學試題

填准考證號碼

第一頁 共一頁

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#### 注意事項：

1. 本試題共四題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

#### Problem 1. (25%)

- a) (15%) Please prove the following convolution integral result

$$L^{-1}\{F(s) \cdot G(s)\} = f(t) * g(t)$$

where  $L^{-1}$  is inverse Laplace transform,  $*$  is convolution integral

$$f(t) = L^{-1}\{F(s)\}, \quad g(t) = L^{-1}\{G(s)\}$$

- b) (10%) Please use the above result to compute the following

$$L^{-1}\left\{\frac{s}{(s^2 + \omega^2)^2}\right\} = ?$$

#### Problem 2. (25%)

- a) (15%) Please find the solution of ordinary differential equation

$$(x^2 + 1)y'' - 2xy' + 2y = 6(x^2 + 1)^2$$

$$\text{Boundary condition } y(x=0) = -1, y(x=1) = 5$$

- b) (10%) Please find the solution of ordinary differential equation

$$\frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx}\right)^3$$

**Problem 3. (25%)**

Please find the following Principal Value Integral (P.V.)

$$P.V. \int_{-\infty}^{\infty} \frac{e^{qx}}{1-e^x} dx = ? \quad \text{for } 0 < q < 1$$

**Problem 4. (25%)**

Please Check or Interpret the following terms from a) to e)

- a) (5%) “Riemann surface” for double roots, “Branch Cut” for the Plane
- b) (5%) What is existence condition of  $f(z)$  by the “Conformal Mapping”?  
for  $z \in C$
- c) (5%) What is existence condition of  $f(t)$  by the “Laplace Transform”?  
for  $0 < t < \infty$
- d) (5%) What is existence condition of  $f(t)$  by the “Fourier Transform”?  
for  $-\infty < t < \infty$
- e) (5%) What is condition of “Hermitian Matrix” of  $A$ ?