

國立臺北科技大學

九十三年學年度電腦通訊與控制研究所入學考試

工程數學(甲乙丁組)試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共 8 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (a) As shown in the following circuit Fig. (a), show that

$$I_1 = \left(\frac{R_2}{R_1 + R_2} \right) I = \left(\frac{R}{R_1} \right) I \quad \text{and} \quad I_2 = \left(\frac{R_2}{R_1 + R_2} \right) I = \left(\frac{R}{R_2} \right) I, \quad \text{where} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \quad (10\%)$$

(b) Determine the unknown currents in the given circuit Fig (b). (10%)

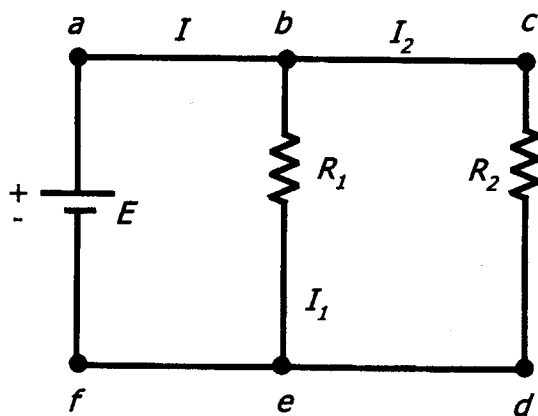


Fig. (a)

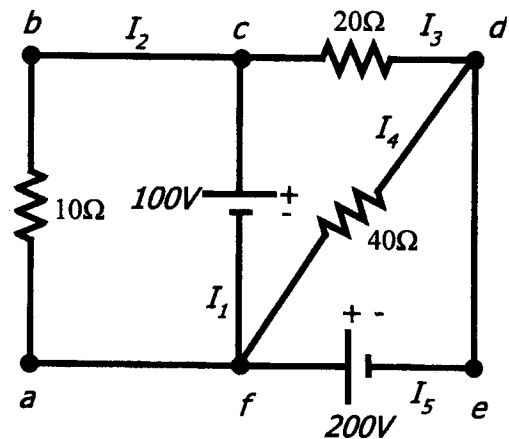


Fig. (b)

2. Show that if A is diagonalizable, then

(a) A^T is diagonalizable. (5%)

(b) A^k is diagonalizable, where k is a positive integer. (5%)

3. Find the general solution to the homogeneous linear system of differential equations. (10%)

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}.$$

4. Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by

$$L \left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

(a) Is L onto? Why (5%)

(b) Find a basis for range L . (5%)

(c) Find $\ker L$. (5%)

5. A die is tossed and the number of dots N_1 is noted; an integer N_2 is then selected at random from $\{1, \dots, N_1\}$.

(a) Find the probability of the event $\{N_2 = 3\}$. (5%)

(b) Find the probability of $\{N_1 = 4\}$ given $\{N_2 = 5\}$. (5%)

6. A random variable X has pdf

$$f_x(x) = \begin{cases} cx(1-x^4) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find c . (5%)

(b) Find the cdf of X . (5%)

(c) Find $P[|X| < 1/2]$. (5%)

7. Let (X, Y) has the joint pdf

$$f_{X,Y}(x, y) = xe^{-x(1+y)} \quad x > 0, y > 0.$$

Find the marginal pdf of X and Y . (10%)

8. Let (X, Y) be the jointly Gaussian random variable discussed in the problem 7. Find $P[X^2 + Y^2 < R^2]$ when $\rho = 0$. Hint: Use polar coordinates to compute the integral. (10%)