國立臺北科技大學

九十三學年度電機工程系碩士班入學考試

工程數學試題（丁組與戊組）

填准考證號碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (8%) A shipment of 10 television sets contains 3 defective sets. In how many ways can a hotel purchase 5 of these sets and receive at least 2 of the defective sets.

2. (15%) The joint probability density function of the random variables X, Y and Z is given by

$$f_{XYZ}(x,y,z)=\begin{cases}4xyz^2/9 & 0<x<1, 0<y<1, 0<z<3 \\ 0 & \text{else}\end{cases}$$

a) (5%) Find the joint marginal probability density function of two random variables Y and Z.

b) (5%) Find the marginal probability density function of random variables Y.

c) (5%) Find the conditional probability $P(0<X<1/2, 0<Z<1 \mid Y=1/2)$.

3. (16%) Let X be a Gaussian random variable with probability density function

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

a) (10%) Show that the moment-generating function of the random variable X is given by

$$M_X(t)=E\left[e^{tx}\right]=e^{\mu t+\frac{1}{2}\sigma^2 t^2}$$

b) (6%) Use $M_X(t)=e^{\mu t+\frac{1}{2}\sigma^2 t^2}$, find $E[X]$ and $E[X^2]$.

4. (11%) The random variables X and Y have the joint density

$$f_{XY}(x,y)=\begin{cases} x+y & 0<x<1, 0<y<1 \\ 0 & \text{else} \end{cases}$$

a) (6%) Find the covariance of random variables X and Y.
b) (5%) Find the covariance of random variables 4X+7 and 9Y-12.

5 Given $x-2y+3z=1$, $2x+ky+6z=6$, and $-x+3y+(k+3)z=0$ (20%)

a). Find "k" such that the equations are ***inconsistent***

b). Find "k" such that there exists a ***unique*** solution for the equations

6 Select the ***wrong*** statement(s) **and** give the ***reason(s) (30%)***

a). Given $F=G+H$ *where* $F \in R^3$, $G \in R^3, H \in R^3$ then $F \bullet (G \times H) = 0$

b). Given $S=\{(x,y,2x,3y)/x,y \in real\}$ then (1,0,2,0) and (0,1,0,3) form a basis for the subspace S of R^4 and the dimension of the subspace equals 4.

c). Given $A_{n\times n}$ and $B_{n\times n}$ then $AB=BA$

d). Given $A_{n\times n}$ then Rank (A) $= n \leftrightarrow A_R = \mathrm{I}_n \leftrightarrow A^{-1}$ exists$\leftrightarrow |A| \neq 0 \leftrightarrow A_{n\times n}$ is non-singular; where " $\leftrightarrow$ " means "if and only if".

e). Given $A \equiv [a_{ij}]_{n\times n}$ *and* $B \equiv [\alpha a_{ij}]_{n\times n}$ then $|B| = \alpha |A|$