國立臺北科技大學

九十三學年度電機工程系碩士班入學考試

工程數學試題(甲組、乙組與丙組)

填准考證號碼 | 第一頁 共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Given $A^2 > 4B$, "A" and "B" are constants
   Show that $y(t) = e^{\alpha t}(c_1 \cosh(\beta t) + c_2 \sinh(\beta t))$ is the general solution of $y'' + Ay' + By = 0$ for appreciate choices of α and β; C_1 and C_2. (20%)

2. Given $f(t) = |E \sin(\omega t)|$, "$E$" and "$\omega$" are positive numbers.

   Find the Laplace Transform of *f(t)*. (15%)

3. Given $f(t) = \begin{cases} 0 & \forall t < 5 \\ t^2 + 2t + 1 & \forall t \geq 5 \end{cases}$

   Find the Laplace **Transform of *f(t)*. (15%)**

4. Given $x - 2y + 3z = 1$, $2x + ky + 6z = 6$, and $-x + 3y + (k+3)z = 0$ (20%)
   (1). Find "k" such that the equations are ***inconsistent***
   (2). Find "k" such that there exists a ***unique*** solution for the equations

5. Select the ***wrong*** statement(s) **and** give the ***reason(s) (30%)***

   (1). Given $F = G + H$ *where* $F \in R^3$, $G \in R^3, H \in R^3$ then $F \bullet (G \times H) = 0$

4. Please prove that:
   (a) A signal $g(t)$ and its Hilbert transform $\hat{g}(t)$ have the same magnitude spectrum.(5%)
   (b) If $\hat{g}(t)$ is the Hilbert transform of $g(t)$, then the Hilbert transform of $\hat{g}(t)$ is $-g(t)$.(5%)

5. A FM signal is defined as $s(t) = A_c \cos(2\pi f_c t + 2\pi \int_0^t m(\tau)d\tau)$. The modulating signal $m(t)$ is defined as below:

   $m(t) = A_m \sin^2(2\pi f_m t)$

   Please find the approximate form of a narrowband($\frac{A_m}{f_m}$ is small compared to one radian) FM signal.(10%)

6. The AM signal is defined as below: (10%)
   $s(t) = A_c[1 + 0.8\sin(2\pi f_m t)]\cos(2\pi f_c t)$
   Please find the value of $\frac{(SNR)_O}{(SNR)_C}$.

7. According to the Nyquist's theorem, to avoid the aliasing effect, please find the minimal sample rate(Hz) for the following signal. (10%)
   (a) $g(t) = \sin c(100\pi t)$
   (b) $g(t) = \sin c^2(100\pi t)$
   (c) $g(t) = \sin c(100\pi t) + \sin c^2(100\pi t)$
   (d) $g(t) = \sin c(100\pi t) * \sin c^2(100\pi t)$

8. The filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive white Gaussion noise $w(t)$, as shown by
   $x(t) = g(t) + w(t), 0 \le t \le T$.
   Please derive the optimal match filter $h_{opt}(t)$.(10%)

9. (a) Please sketch the waveforms of the in-phase and quadrature components of the MSK signal in response to the input binary sequence 10101100011. (5%)
   (b) Please sketch the MSK waveform itself for the binary sequence specified in part (a). (5%)