

# 國立臺北科技大學

## 九十三年度資訊工程系碩士班入學考試

### 離散數學試題

填准考證號碼

第一頁 共二頁

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#### 注意事項：

1. 本試題共 12 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Let  $A = \{1, 2, 3, \dots, 600\}$ . Find the number of positive integers in  $A$  that are not divisible by 3 or 5 or 7? (10%)
2. Prove the following:
  - (1)  $m \binom{m-1}{r-1} = r \binom{m}{r}$  (5%)
  - (2)  $\binom{m+1}{r+1} = \sum_{k=r}^m \binom{k}{r}$  (5%)
3. Let  $@$  be a binary operation defined on the set of integers  $Z$  by  $x @ y = x+y-5$ . Determine whether or not  $(Z, @)$  is an abelian group. (5%)
4. For  $n \in Z^+$  define the sum  $s_n$  by the formula
$$s_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n-1}{n!} + \frac{n}{(n+1)!}$$
  - (1) Compute  $s_1, s_2, s_3, s_4$  and  $s_5$ . (5%)
  - (2) On the basis of your results in part (1), conjecture a formula for the sum of the terms in  $s_n$ . (5%)
  - (3) Use mathematical induction to verify your conjecture in part (2) for all  $n \in Z^+$ . (5%)

5. There is a radar, a computer, and a gyroscope on board an airplane. The probability that the radar fails is 0.2. If the radar fails, the gyroscope will also fail, and the possibility that the computer fails is 0.3. If the radar functions correctly, then the computer will also function correctly, and the probability that the gyroscope fails is 0.2.
- (1) Describe the sample space. (3%)
  - (2) What is the probability that the computer or the gyroscope functions correctly while the other does not? (3%)
  - (3) What is the probability that the radar functions correctly if one of the other two systems fails? (4%)
6. A binary tree  $T$  is *balanced* if for every vertex  $v$  in  $T$ , the heights of the left and right subtrees of  $v$  differ by at most 1. We define the height of an empty tree to be -1. Let  $N_h$  be the minimum number of vertices in a balanced binary tree of height  $h$  and  $f_1=1, f_2=2, \dots$  denote the Fibonacci sequence. Please show that  $N_h=f_{h+2}-1$  for  $h \geq 0$ . (10%)
7. Please represent the following expression as a binary tree (2%) and write the prefix (4%) and postfix (4%) form of the expression.
- $$(((A+B)*C+D)*E)-((A+B)*C-D)$$
8. Find the values of the following postfix expressions if  $A=1, B=2$ , and  $C=3$ .
- (1)  $ABC+-$  (3%)
  - (2)  $ABC**ABC++-$  (2%)
9. Figure 1 shows a finite state machine. Please construct a grammar that specifies the language accepted by the finite state machine. (10%)

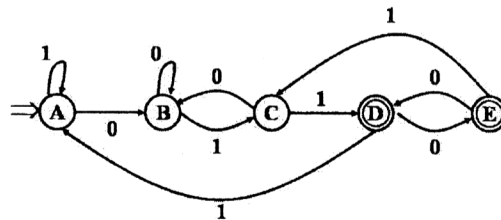


Figure 1. A finite state machine

注意：背面尚有試題

10. Please simplify the following expression

$$(p \wedge [(q \wedge r) \vee (q \wedge r \wedge \neg p)]) \vee (p \wedge q \wedge \neg r)$$

to an expression having as few symbols as possible. (5%)

11. True or false (1% each)

(1)  $x^2 = O(x^3)$

(2)  $x \log_2 x = \Omega(x^2)$

(3)  $x^{1/2} = O(\log_2 x)$

(4)  $x^{-1} = O(\log x)$

(5)  $\log_2 x = O(x^{-1/2})$

12. Solve the following recurrence relation:

$$T(n) = \sqrt{n}T(\sqrt{n}) + cn \text{ for } n > 2 \text{ and } T(2) = 1,$$

where  $c$  is some positive constant. (5%)