國立臺北科技大學

九十二學年度機電科技研究所博士班入學考試

工程數學（車輛組）試題

注意事項：

1. 本試題共 3 題，配分共 100 分，第一題 30 分，第二題 30 分，第三題 40 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. A 0.5 kg mass (4.905 kg weight) is suspended from a spring, stretching it 0.22295 m. Then the entire system (weight and spring) is submerged in a fluid that imposes a drag force (N) of 2v (v is velocity of the weight), and the system rests in the fluid. The mass is then pulled down $\frac{2}{26}$ m and released. At the same time, the entire system is subjected to an external force 4cos(2t). If buoyancy is neglected, describe displacement of the mass as a function of time.

2. Consider a system

$$\dot{x}_1 = x_1 - x_2 + 4x_3$$
$$\dot{x}_2 = 3x_1 + 2x_2 - x_3$$
$$\dot{x}_3 = 2x_1 + x_2 - x_3$$

Such system can be represented by a state space form as $\dot{X} = AX$.

(a) For convenience of designing a controller, this system must be converted into an uncoupled system $\dot{\hat{X}} = \hat{A}\hat{X}$. Show the detailed steps of finding such an uncoupled system. Also find the new converted state variable $\hat{X}$.

(b) find A^{11}

3. A simple suspension system has parameters m=1, k=6, and c=5.   This suspension system

is subjected to an external force $f(t) = \begin{cases} t^2 & 0 \le t < 1 \\ 1 & 1 \le t < 4 \\ 0 & 4 \le t \end{cases}$.
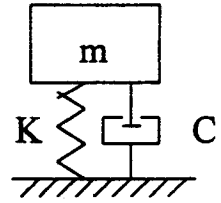
(a) find Laplace transform of this external force;

(b) write the Fourier cosine series of this function for the interval $0 \le t < 4$.

(c) what values will this Fourier cosine series converge to in those periods?

$(\int x^n \cos ax \, dx = \dfrac{x^n}{a} \sin ax - \dfrac{n}{a} \int x^{n-1} \sin ax \, dx, \quad \int x^n \sin ax \, dx = -\dfrac{x^n}{a} \cos ax + \dfrac{n}{a} \int x^{n-1} \cos ax \, dx)$